Roll No.

Total No. of Questions: 09 **Total No. of Pages: 02**

B. Tech. (Sem. 2)
ENGINEERING MATHEMATICS-II
Subject Code: BTAM-102
Paper ID: A1111

Time: 3 Hrs. **Max. Marks: 60**

INSTRUCTIONS TO CANDIDATES:

1. **Section A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **Attempt FIVE questions from Section B & C selecting at least two from each section. All questions carry EIGHT marks.**

SECTION A

l. (a) Find the value of a so that the differential equation $xy^3\,dx + ax^2\,y^2\,dy = 0$ is exact.

(b) Find the solution of the differential equation $y''' - 3y' - 2y = 0$.

(c) Write the differential equation describing the motion of bob of a simple pendulum.

(d) Solve the differential equation $x^2\,y'' + 2xy' - 2y = 0$.

(e) Find the value of Z for which e^z is real.

(f) Examine whether the vectors are linearly independent (2, 2, l) (l,-1,l) (l,0, l).

(g) State integral test for convergence of infinite series.

(h) Define unitary matrix.

(i) Write Bernoulli's equation.

(j) Sow that the geometric series $\sum_{n=0}^{\infty} r^n$. where r is any real number is convergent For $|r| < 1$.

SECTION B

2. (a) Find the solution of the differential equation $(3x^2\,y^3e^y + y3 + y2)\,dx + (\,x^3\,y^3e^x - xy\,)\,dy = 0$.

(b) The initial value problem governing the current i flowing in a series RL circuit, when a voltage $v(t) = t$ is applied, is given by $iR + L\frac{di}{dt} = t,\ t \geq 0,\ i(0) = 0$.

3. Find the general solution of the equation $y'' + 16y = 32\sec 2x$, using the method of variation of parameters.

4. A simple pendulum of length l is oscillating through a small angle Θ in a medium in which the resistance is proportional to velocity. Find the differential equation of its motion. Discuss the motion and find its period of oscillation.

5. (a) Find the general solution of the equation $x^2 y'' + 5xy' + 3y = \text{In } x, x > 0$.

(b) Solve $y = 2xp + y^2 p^3$, where $p = \frac{dy}{dx}$.

SECTION C

6. (a) Examine whether the following matrix is diagonalizable. If so, obtain the matrix P such that $p^{-1} AP$ is a diagonal matrix

$$A=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

(b) Let T be a linear transformation defined by

$$T\left[\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\right]= \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, T\left[\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}\right]= \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, T\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}\Big]= \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}, Ti\left[\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right]= \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix},$$

Find $T\left[\begin{pmatrix} 4 & 5 \\ 3 & 8 \end{pmatrix}\right]$

7. (a) Discuss the convergence of the series $\Sigma \frac{1.4.7.....(3n-2}{2.5.8.....(3n-1}$

(b) Discuss the convergence of the series Σa_n, where $a_n = \left(1+\frac{1}{n^p}\right)-n^{p+1}$, $p>0$, using Cauchy's root test.

8. (a) Using De Moivre's theorem, show that $\cos^4 \Theta = \frac{1}{8}(\cos 4\Theta + 4\cos 2\Theta + 3)$.

(b) Write $\mathbf{tan}^{-1} z$ in the form $u+iv$.

9. (a) Find the Eigen values and corresponding Eigen vector of the matrix $A \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$

(b) Find all values of Z, such that $e^z = 1 + i$.