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Total No. of Questions: 09 **Total No. of Pages: 02**

B. Tech. (Sem. 2)

ENGINEERING MATHEMATICS - II

Subject Code: AM-102

Paper ID: A0119

Time: 3 Hrs. **Max. Marks: 60**

INSTRUCTIONS TO CANDIDATES:

1. **Attempt all sub-questions from Question Sections A (2 Marks each)**
2. **Attempt any FIVE questions from Sections B and C, selecting at least 2 from each (8 Marks each)**

SECTION A

1.

a) Suppose that a matrix A is both unitary and Hermitian. Then show that A = A^{-1}.

b) Show that the eigenvalues of a skew-Hermitian matrix are either zero or purely imaginary numbers.

c) Find general solution of the Clairaut equation $y = xy' - (y')^3$.

d) Find homogeneous linear differential equation with real constants coefficients of lowest order which has $xe^{-x} + e^{2x}$ as a particular solution.

e) Find general solution of $4x^2y" + 8y' + 17y = 0$.

f) Prove that div*(f v) = f(div v)* + (grad *f*)• v, where *f* is a scalar function.

g) State Fisher's z-test.

h) State Divergence Theorem of Gauss.

i) The continuous random variable X has probability density function *(pdf)*

$$f(x) = \begin{cases} \frac{x}{2}, & 0 < x < 2 \\ 0, & elsewhere \end{cases}$$. Find the *pdf* of Y = 3X + 2

j) Discuss χ^2 test for goodness of fit.

SECTION B

2.

(i) State and prove Cayley Hamilton Theorem.

(ii) Check the system of linear equations given by $\begin{bmatrix} 1 & -4 & 7 \\ 3 & 8 & -2 \\ 7 & -8 & 26 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ 3 \end{bmatrix}$ is consistent or not. Consistent then find solution of the system.

3. If $\frac{\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)}{N} = f(x)$, a function of x alone then show that $e^{\int f(x)dx}$ is an integrating factor of M(x, y)dx + N(x, y)dy = 0. Hence solve $(5x^3 + 12x^2 + 6y^2)dx + 6xydy = 0$.

4. (i) Find the general solution of the equation $y" + 16y = 32 \sec 2x$, using the Method of Variation of Parameters.

(ii) Find the general solution of the equation $y'' + 16y = 16\sin 4x$, by the Method of Undetermined Coefficients.

5. An electric circuit consists of an inductance of 0.1 henry, a resistance of 20 ohms and a condenser of capacitance 25 micro-farads. Find the charge q and the current I at any time t, given that at t = 0, q = 0.05 coulomb, i = $\frac{dq}{dt} = 0$ when t = 0

SECTION C

6. (i) Evaluate the line integral of $v = x^2\, i - 2y\, j + z^2 k$ over the straight line path from (- 1, 2, 3) to (2, 3, 5).

(ii) Show that the vector field $F = 2x\,(y^2 + z^3)i + 2x^2 yj + 3x^2 z^2 k$ is conservative. Find its scalar potential and the work done in moving a particle from (-1, 2, 1) to (2, 3, 4).

7. Verify the Green's Theorem for *f(x, y)* = $e^{-x}\sin y, g\,(x, y) = e^{-x}\cos y$ and C is the square with vertices at (0, 0), (π /2, 0), (π /2, π/2), (0, π/2)

8. Show that the Normal distribution is the limiting form of Binomial distribution.

9. The annual rainfall at a certain place is normally distributed with mean 45cm. The rainfall during the last five years are 48cm, 42cm, 40cm, 44cm and 43cm. Can we conclude that the average rainfall during the last five years is less than the normal rainfall? Test at 5% level of significance.