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Total No. of Questions: 09 **Total No. of Pages: 02**

B. Tech. (Sem. 1)

ENGINEERING MATHEMATICS-I

Subject Code: AM-101

Paper ID: A0111

Time: 3 Hrs. **Max. Marks: 60**

INSTRUCTIONS TO CANDIDATES:

1. **Attempt all sub-questions from Question 1 (2 Marks each)**
2. **Attempt any FIVE questions from Sections A and B, selecting at least 2 from each Section (8 Marks each)**

SECTION A

1.

a) Find radius of curvature of the curve given by x = a In (sec t + tan t), y = a sect.

b) Find the length of the curve y = 1n(e^x + 1) - 1n(e^x - 1) from x = 1 to x = 2.

c) Show that the function $f(x,y)\begin{cases}\frac{xy}{x^2+2y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$ is not continuous at (0, 0) but its partial derivatives f_x and f_y exist at (0, 0).

d) Let f(x, y) and g(x, y) be two homogeneous functions of degree m and n respectively, where $m \neq 0$, Let $h = f + g$, if $x\frac{\partial h}{\partial x}+y\frac{\partial h}{\partial y}=0$ then show that $f = \alpha\, g$ for some scalar .

e) State sufficient condition for a function of two variable to have maximum or minimum.

f) Find the value of k such that x2 + y2 + z2 + 2x - 4y + 6z + k = 0 represents a sphere of radius 5.

g) Show that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m)\Gamma(n)}$

h) Define uniform convergence of infinite series.

i) Prove that i^i is wholly real and find its principal value.

j) Find the value of $(\sqrt{3}+i)^{\frac{1}{3}}$. 1

SECTION B

2. Trace the curve $y=\frac{x^2-3x}{(x-1)}$

3.

a) What is the volume generated by revolving the area enclosed by the loop of the curve y4 = x (4 - x) about the x-axis.

b) Find the area of the surface of the solid of revolution generated by revolving the parabola y2 = 4ax, $0 \leq x \leq 3a$ about the x-axis.

4.

a) If z = 1n(u 2 + v), $u = e^{x+y^2}$, $v = x + y^2$ then show that $2y\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}=0$

b) Using Euler's theorem show that if $u = \sqrt{y^2 - x^2}\, sin^{-1}\left(\frac{x}{y}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial r}{\partial y} = u.$

5.

a) State and prove Taylor's theorem for function of two variable.

b) A rectangular box without top is to have a given volume. How should the box be made so as to use the least material.

SECTION C

6. Obtain the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9$, $x + y + z = 3$ as a great circle. Also find its centre and radius.

7.

a) Evaluate $\iiint_T \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2}}\, dx\, dy\, dz,$ the boundary of $T = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$

b) Using Beta and Gamma functions, evaluate $\int_{-1}^{1}(1 - x^2)^n\, dx,$ where n is a positive integer.

8.

a) Discuss the convergence of the series $\sum \frac{(\angle n)^2}{\angle 2n} x^n, x > 0$

b) Find the values for which the power series $\sum \frac{(x+2)^n}{n^2}$ converges.

9.

a) Apply De moivre's theorem to find an equation whose roots are the nth powers of the roots of the equation $x^2 - 2x\cos\theta + 1 = 0.$

b) Sum ton terms the series $sin\,\theta + \frac{1}{3} sin\,2\theta + \frac{1}{3^2} sin\,3\theta + ...$