December 2005
CS-203
ENGINEERING MATHS -III
(B.Tech., 3rd/4th Semester, 2125)

Time : 03 Hours **Maximum Marks : 60**

Note : Section - A is compulsory. Attempt any **Four questions from** section B. Attempt any **Two** questions from section C.

Section A **(Marks : 2 Each)**

Q.1. (a) Find all values of z such that $\sin z = 2$.

(b) Evaluate the following integral $\oint_C \frac{3z+5}{z^2+2z}dz, C:|z|=1$

(c) Show that the function $\frac{\sin z}{z^r}, r \geq 2$ has a pole of order [illegible] at $z=0$

(d) Find the Fourier cosine and sine series of the funciton $f(x)$ [illegible] ≤ 2.

(e) State the sufficient condtions for the existence of the L[illegible] of a function. Also, write a function whose Laplace transform exists but it violates the [illegible]itions.

(f) Evaluate the following integral using Laplace transform [illegible] dt.

(g) Find the Fourier transform of the function $f(t), f(t)=e^{-a|t|}, -\infty<t<\infty, a>0$, write the inverse transform.

(h) Classify the following partial differential equation $(1-y)\frac{\partial^2 u}{\partial x^2}+2x\frac{\partial^2 u}{\partial x \partial y}+(1+y)\frac{\partial^2 u}{\partial y^2}=0$.

(i) Find a relationship between Laplace transform and Fourier transform.

(j) Write the Range-Kutta fourth order method for the following second order differential equation.

$$\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=R(x), y(x_0)=y_0 \ and \left(\frac{dy}{dx}\right)_{x=x_0}=z_0 \text{ and } P(x), Q(x), R(x)$$

are continuous functions.

Section B **(Marks : 5 Each)**

Q.2. If $f(z) = u(x, y) + iv(x, y)$ is an analytic function of $z=x+iy$ and $u+v=(x+y)(2-4xy+x^2+y^2)$ then find u, v and the analytic funtion $f(z)$

Q.3. Find the Fourier series expansion of the following periodic function of period 4

$$f(x)=\begin{cases} 2+x, & -2\leq x\leq 0 \\ 2-x, & 0<x\leq 2 \end{cases} \quad f(x+4)=f(x).$$

Q.4. Find the solution of the initial value problem $y''+ty'-2y=6-t, y(0)=0, y'(0)=1$, using Laplace transform method. Given that L {y(t)} exists.

Q.5. Solve the following partial differential equation $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x \partial y}-6\frac{\partial^2 z}{\partial y^2}=y\cos x$.

Q.6. Show that there are several Runge-Kutta methods and one of them is Euler's improved method.

Section C **(Marks : 1**

Q.7. Show that the transformation $\omega = e^z$ is always conformal. Under this mapping, find the images ragions.

(i) the line segment $0 < y < A, A < 2\pi, x < 0$

(ii) the rectangle bounded by the lines $x = 0, y = 0, x = 1$, and $y = \pi$

Q.8. (a) Write the Filtering property of Dirac-Delta function. Also, find the Laplace transform of Dir function.

(b) Using the Fourier integral transform, solve the following initial value problem

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, -\infty < t < \infty, t > 0, u(x,0) = e^{-2|x|}, -\infty < t < \infty.$$

Q.9. (a) Establish the wave equation in one dimension.

(b) Using Adams- Bashforth method, compute y (0.3) of differential equation.

$\frac{dy}{dx} + \frac{1}{10} y^2 = x$, which satisfies the following sets of values of x and y :

x	-0.2	-0.1	0.0	0.1	0.2
y	1.04068	1.01513	1.0	0.99507	1.00013