May 2005
CS-204
MATHEMATICS -III
(B.Tech., 4th Semester, 2055)

Time : 03 Hours **Maximum Mark**

Note : Section - A is **compulsory.** Attempt any **Four** questions from section B. Attempt any **Two** ques from section C.

Section A
(Marks: 2 Ea

Q.1 (a) Evaluate $\int_0^1\int_0^x e^{x^2}\,dy\,dx$.

(b) Show that an analytic function with constant modulus is constant.

(c) Evaluate $\int_C \frac{z^2-3z+4}{z-2}dz$, where $C:|z|=3$

(d) Expand $1/z$ by Taylor series about the point $z=1$.

(e) Define a bilinear transformation.

(f) Explain briefly the Picard's method of successive approximations.

(g) Find $y(0.1)$ by Euler's method, where $\frac{dy}{dx}=y-2x^2; y(0)=1$

(h) Runge-Kutta method is better than Taylor's series method. Comment.

(i) Solve by the method separation of variables, $py^3+qx^2=0$

(j) An infinitely long plane uniform plate is bounded by two parallel edges and an end at right angl them. The breadth is π; this end is maintained at a temperature u_0 at all points and other ed are at zero temperature. If it is required to find the temperature at any point of the plate in the ste state, then write the relevant boundary conditions for this problem.

Section B
(Marks: 5 Eac

Q2. Evaluate $\int\int \frac{r\,d\theta\,dr}{\sqrt{a^2+r^2}}$ over one loop of the lemniscate $r^2=a^2\cos 2\theta$

Q3. Show that the function $|z|^2$ continuous at every point but is not differentiable at any point other th the origin.

Q4. Expand $\frac{1}{(z^2+1)(z^2+2)}$ as a Laurent series valid for :

$(a)\ 0<|z|<1$ $\quad (b)\ 1<|z|<\sqrt{2}$ $\quad (c)\ |z|>\sqrt{2}$

Q5. Solve the equation, $\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}$ with the conditons $u(0,t)=0$, $u(x,0)=x(1-x)$, and $u(1,t)=$ Assume $h=0.1$ and choose suitable k so that $u(i,j)$ is found out for $i=0, 0.1, 0.2$ ....$0.9, 1$ an $j=k, 2k, 3k$.

Q6. Using Modified Euler's method, find an approximate value of y when $x=0.2$, given that $\frac{dy}{dx}=x+y$ and $y=1$ when $x=0$

Section C **(Marks: 10 Each)**

Q7. (a) Find the analytic function whose imaginary part is $\frac{x-y}{x^2+y^2}$

(b) By Contour integration, show that: $\int_0^\infty \frac{1-\cos x}{x^2}\,dx = \frac{\pi}{2}$

Q8. Find the solution of the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$, corresponding to the triangular initial deflection :

$$f(x) = \begin{cases} \frac{2k}{l}x, & \text{when } 0 < x < \frac{l}{2} \\ = \frac{2k}{l}(l-x), & \text{when } \frac{l}{2} < x < l \end{cases}$$ and initial **velocity zero.**

Q9. Using Runge-Kutta method of order 4, solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ **wtih $y(0)=1$ at** $x = 0.2, 0.4$.