

May-2007

(CS-203)

MATHEMATICS-III

(B.Tech 4th Semester, 2057)

Time : 03 Hours

Maximum Marks : 60

Note : Section-A is compulsory. Attempt any Four questions from section - B. Attempt any two questions from Section C.

Section A

- Q. 1. (a) Discuss the analyticity of $\cos z$.
(b) State Cauchy's Mean Value Theorem.
(c) Show that the real and imaginary parts of an analytic function satisfy the Laplace equation.

(d) Evaluate $\int_0^{1+i} z^2 dz$ along the path $x = 3y^2$.

(e) Show that Euler's Method is a Runge-Kutta method of first order.

(f) Classify the following partial differential equation;

$$2 \frac{\partial^2 u}{\partial x^2} + 4 \frac{\partial^2 u}{\partial x \partial y} + 3 \frac{\partial^2 u}{\partial y^2} = 0$$

(g) Find the image of the line $y - x + 1 = 0$ under the mapping $w = \frac{1}{z}$.

(h) Define Boundary and Initial conditions.

(i) Find the residue of $z \cos \frac{1}{z}$ at $z = 0$.

(j) Find the inverse Laplace transform of $\frac{1}{s(s-1)}$.

Section B

Q. 2. Express the area between the curves $x^2 + y^2 = 3^2$ and $x + y = 3$ as a double Integral and evaluate it.

Q. 3. Use residue Calculus to evaluate $\int_0^{2\pi} \frac{1}{5 - 3 \sin \theta} d\theta$.

Q. 4. Use Taylor's series method to solve the equation $\frac{dy}{dx} = -xy$; $y(0) = 1$.

Q. 5. Solve the equation $\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by the method of separation of variables.

Q. 6. Find the mapping of real-axis under the transformation $w = \frac{z-i}{z+i}$ on to the w -plane.

Section C

Q. 7. Find the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ subject to the conditions;

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(a) u is not infinite when $t \rightarrow \infty$.

(b) $\frac{\partial u}{\partial x} = 0$ for $x = 0$ and $x = l$.

(c) $u = lx - x^2$ for $t = 0$ between $x = 0$ and $x = l$.

Q. 8. (a) Solve the equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0 = y; x =$ with $u = 0$ on the boundary and mesh length = 1.

(b) Apply Runge-Kutta fourth order method to find an approximate value of y when $x = 0.2$ give

$$\frac{dy}{dx} = x + y \text{ and } y = 1 \text{ when } x = 0.$$

Q. 9. (a) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions;

$$u(0, y) = u(l, y) = u(x, 0) = 0 \text{ and } u(x, a) = \sin \frac{n\pi x}{l}$$

(b) Apply Calculus of Residues to prove that $\int_0^\infty \frac{dx}{(x^2 + a^2)^3} = \frac{\pi}{4a^3}; a > 0$.