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Total No. of Questions : 09]

[Total No. of Pages : 03

# B.Tech. (Sem. - 4th)

## MATHEMATICS - III

**SUBJECT CODE : CS - 204**

**Paper ID : [A0495]**

[Note : Please fill subject code and paper ID on OMR]

**Time : 03 Hours** **Maximum Marks : 60**

**Instruction to Candidates:**

1) Section - A is **Compulsory**.
2) Attempt any **Four** questions from Section - B.
3) Attempt any **Two** questions from Section - C.

## Section - A

**Q1)** *(10 × 2 = 20)*

a) Show that the function $f(x) = \sin\left(\frac{1}{x}\right)$ is continuous & bounded on $\left(0, \frac{2}{\pi}\right)$ but is not uniformly continuous there.

b) State necessary & sufficient condition for $f(z)$ to be analytic.

c) Write Cauchy-Riemann equation in Polar co-ordinates.

d) Evaluate $\oint_C (x^2 - y^2 + 2ixy)dz$, where C is the contour $|z| = 1$.

e) Classify the partial differential $= u$
$(x + 1)\, u_{xx} - 2(x + 2)\, u_{xy} + (x + 3)\, u_{yy} = \cos(x - 2y)$

f) Find the Laplace transform of $f(t)$ defined as

$$f(t) = \begin{cases} \frac{t}{T}, \text{ when } 0 < t < T \\ 1, \text{ when } t > T. \end{cases}$$

g) State convolution theorem for Fourier Transform.

h) Discuss applicability for Rolle's theorem to the function $f(x)$ defined as follows :

$$f(x)=\begin{cases} x^2-4, \text{ if } x<1 \\ 5x-8, \text{if } x\geq 1 \end{cases} \text{ in } \left[-2,\frac{8}{5}\right]$$

i) Define unit Impulse function & find $\int_0^\infty t^3\ \delta(t-5)$.

j) Define periodic function with examples.

## Section - B

*(4 × 5 = 20)*

*Q2)* Use Cauchy's Integral Formula to evaluate $\oint_C \frac{e^{2z}}{(z+1)^4}dz$, where C is the circle $|z|=2$.

*Q3)* Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(1+x^2)^3}dx$.

*Q4)* Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$, by changing to Spherical polar co-ordinates.

*Q5)* An electrostatic field in the $xy$-plane is given by the potential function $\phi=3x^2y-y^3$, find the stream function.

*Q6)* Find a series of cosines of multiples of $x$ which will represent $x\sin x$ in the interval $(0,\pi)$ & show that $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\ldots\ldots\ldots=\frac{\pi-2}{4}$

## Section - C

*(2 × 10 = 20)*

*Q7)* Expand $\frac{1}{(z^2+1)(z^2+2)}$ as a Laurent series valid for

(a) $0<|z|<1$

(b) $1<|z|<\sqrt{2}$

(c) $|z|>\sqrt{2}$.

**Q8)** A string is stretched & pastened to two points $l$ apart. Motion is started by displacing the string in the form $y = a\sin\frac{\pi x}{l}$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance $x$ from one end at the $t$ is given by $y(x,t) = a\sin\frac{\pi x}{l}.\cos\frac{\pi ct}{l}$.

**Q9)** Evaluate $y(0.2)$ by Runge-Kutta ($4^{th}$ order) method given that
$y'' - xy'^2 + y^2 = 0;\ y(0) = 1, y'(0) = 0$

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