Roll No. **Total No. of Pages : 03**

Total No. of Questions : 09

B.Tech. (CSE / IT) (Sem.–4th)

MATHEMATICS-III

Subject Code : CS-204

Paper ID : [A0495]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

1. Answer briefly :

(a) Define the term "Uniform continuity".

(b) Verify Rolle's theorem for $f(x) = (x - a)^m (y - b)^n$ where m, n are the integers in $[a, b]$.

(c) State the fundamental theorem of integral calculus.

(d) Define a conformal mapping.

(e) Explain the term "Residue" at a pole.

(f) State clearly the wave equation and the heat equation.

(g) A tightly stretched string of length l with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $v_0 \sin^3 \frac{\pi x}{l}$. If it is required to find the displacement $y(x, t)$ then write the initial and boundary conditions for the problem.

(h) In the equation, $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, what is represented by c^2 ?

(i) Classify the equation, $(1+x^2)\frac{\partial^2 u}{\partial x^2}+(5+2x^2)\frac{\partial^2 u}{\partial x \partial t}+(4+x^2)\frac{\partial^2 u}{\partial t^2}=0$.

(j) Write the 5-point diagonal formula.

SECTION-B

2. Find the area common to the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

3. State and prove the Cauchy's integral formula.

4. Show that $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta}\, d\theta = \frac{\pi}{12}$.

5. Show that the transformation $\omega = \frac{1}{z}$ transforms the hyperbola $x^2 - y^2 = 1$ is a lemniscate.

6. Use Runga-Kutta method of fourth order to solve,

$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$.

SECTION-C

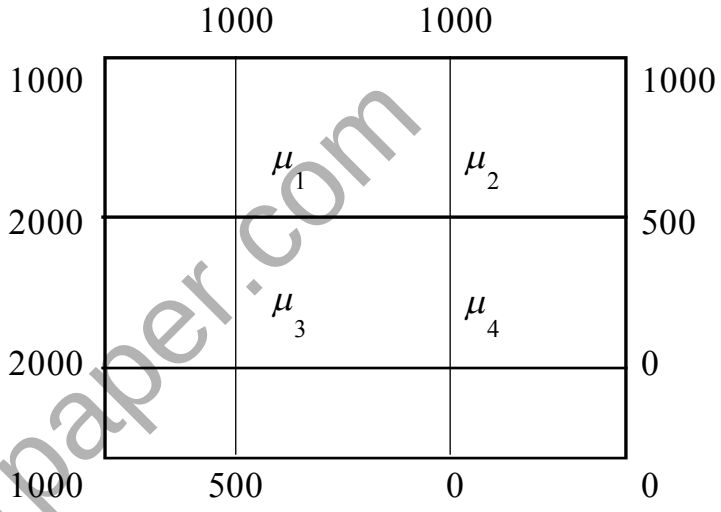
7. If $f(z) = u + iv$ is an analytic function, prove that,

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log|f(z)| = 0$$

8. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions

$\mu(0, y) = \mu(l, y) = \mu(x, 0) = 0$ and $\mu(x, a) = \frac{\sin n\pi x}{l}$.

9. Given the values of μ (x, y) on the boundary of the following square,



Evaluate the function μ (x, y) satisfying the Laplace equation, $\nabla^2\mu = 0$ at the pivotal points of this figure.