

Mathematics III
Subject code:CS-204

B. Tech.

A0495

Section - A

Q. 1:

- (i) Verify Rolle's Theorem for $f(x) = x(8 - x)^m$ $[0, 8]$
- (ii) Evaluate $\int_0^1 \int_0^{x^2} e^y dy dx$
- (iii) Evaluate $\oint_C \frac{z dz}{(z-1)(z-3)}$ value $C: |z| = \frac{3}{2}$
- (iv) Find the poles and residues at each pole of function 'cot'
- (v) Find the image of line $y - x + 1 = 0$ under the mapping $w = \frac{1}{z}$
- (vi) A string of length 'e' is initially at rest in equilibrium position and each of its points is given two velocity $v(x)$ write down boundary and initially conditions.
- (vii) If $L[f(t)] = F(s)$, i.e Laplace transform of $f(t)$ is $F(s)$.
 $f(0) = 1, f'(0) = -1$ find $L(f''(t))$
- (viii) Find fourier transform of $e^{-|x|}$
- (ix) Using Taylor's series method compare the solution of $\frac{dy}{dx} = y^2 + x, y(0) = 1$ at $x = 0.1$
- (x) State diagonal five point formula for finding value of U at any interior mesh point.

Section : B

Q. 2 : Find the volume bounded by the xy plane, the Cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 3$

Q. 3: If $f(z)$ is analytic function f_z prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |Ref(z)|^2 = 2|f'(z)|^2$

Q. 4: Find the transformation which maps the points $-1, i, 1$ of the z plane onto $1, i, -1$ of the w plane

Q. 5: A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is

$$u(x, 0) = \begin{cases} x, & 0 \leq x \leq 50 \\ 100 - x, & 50 \leq x \leq 100 \end{cases}$$

Find the temperature $U(x, t)$ at any time.

Q. 6: Using Range - Kutta method of 4th order find y at $x = 1.1$ given that $y^1 = x^2 + y^2$, $y(1) = 1.5$

Section - C

Q. 7: Evaluate $\int_0^{2\pi} \frac{d\theta}{1 - 2p \cos \theta + p^2}$, $0 < p < 1$ by contour integration

Q. 8: A tightly structured string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position. Find the displacement $y(x, t)$.

Q. 9: Solve $\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$ by Leibman's method over the square mesh of side Four units satisfying the followings boundary conditions

$$(i) U(0, y) = 0 \text{ for } 0 \leq y \leq 4 \quad (ii) U(4, y) = 12 + y, 0 \leq y \leq 4$$

$$(iii) U(x, 0) = 3x, 0 \leq x \leq 4 \quad (iv) U(x, 4) = x^2, 0 \leq x \leq 4$$

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