Roll No. .... **Total No. of Pages : 02**

Total No. of Questions : 09

B.Tech. (CSE/IT) (Sem.–4th)
MATHEMATICS-III
Subject Code : CS-204
Paper ID : [A0495]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTION TO CANDIDATES :

1. **SECTION-A is COMPULSORY.**
2. **Attempt any FOUR questions from SECTION-B.**
3. **Attempt any TWO questions from SECTION-C.**

SECTION-A **(10 × 2 = 20 Marks)**

1. (a) State Cauchy's mean value theorem.

(b) Evaluate $\iint (x + y)\, dy\, dx$ over the region bounded by $x = 0$, $y = 0$, $x^2 + y^2 = 9$.

(c) Evaluate $\int_0^{1+i} (x^2 - iy)\, dz$ along the path, $y = x^2$.

(d) Expand $\sin z$ by Taylor's series about the point $\frac{\pi}{4}$.

(e) For the conformal transformation $w = z^2$, find the angle of rotation at $z = 1 + i$.

(f) A rod of length l with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to 0°C and are kept at that temperature. If it is required to find the temperature function $u(x,t)$ then write down the initial and the boundary conditions.

(g) State the fundamental theorem of integral calculus.

(h) Use Picard's method to solve the equation, $\frac{dy}{dx} = x + y^2$, given that $y(0) = 0$.

(i) Write the formulae for the Runge-Kutta method of order 4.

(j) Determine the residue at the pole of order 2 for the function

$$f(z) = \frac{z^2}{(z-3)(z-2)^2}$$

SECTION-B **(4 × 5 = 20 Marks)**

2. Obtain the value of the triple integral $\iiint (x + y + z)dx\, dy\, dz$ over the tetrahedron bounded by the co-ordinate planes and the plane, $x + y + z = 1$.

3. State and prove the Cauchy's integral formula.

4. Expand, $f(z) = \frac{1}{(z+1)(z+3)}$ valid for (*i*) $1 < |z| < 3$, (*ii*) $0 < |z + 1| < 2$.

5. Use Taylor series method to solve $\frac{dy}{dx} = x^2 - y$ at $x = 0.1$, given that $y(0) = 1$.

6. Solve the partial differential equation, $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$, the boundary conditions are $u(0,t) = 0$, $u(l,t) = 0$ (t >0) and initial condition is $u(x,0) = x$, l being the length of the bar.

SECTION-C **(2 × 10 = 20 Marks)**

7. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = a \sin \frac{\pi x}{l}$ from which it is released at time t = 0. Show that the displacement of any point at a distance x from one end at time t is given by, $y(x,t) = a \sin \frac{\pi x}{l} \cos \frac{\pi ct}{l}$

8. Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx$, (a > 0, b> 0), using the concept of contour integration.

9. Find the values of $u(x,t)$ satisfying the parabolic equation, $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ and the boundary conditions $u(0,t) = 0 = u(8,t)$ and $u(x,0) = 4x - \frac{x^2}{2}$ at the points $x = i : i = 0,1,2,\ldots,7$ and $t = \frac{1}{8} j : j = 0,1,2, \ldots,5$.