

Roll No.....

Total No. of Questions : 09]

[Total No. of Pages : 02

Paper ID [CS203]

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B.Tech. (Sem. - 3rd/4th)

MATHEMATICS - III (CS - 203)

Time : 03 Hours

Maximum Marks : 60

Instruction to Candidates:

- 1) Section - A is **Compulsory**.
- 2) Attempt any **Four** questions from Section - B.
- 3) Attempt any **Two** questions from Section - C.

Section - A

Q1)

(10 × 2 = 20)

- a) State Taylor's Expansion.
- b) State and prove second shifting property of Laplace transforms.
- c) Find the inverse Laplace transform of $2s/4s^2 + 16$.
- d) State Cauchy's integral theorem.
- e) Show that function $|z|^2$ is not analytic at any point.
- f) Write down one dimensional, two dimensional heat flow equations.
- g) Show that if $|z+1| < 1$, $z^{-2} = 1 + \sum_{n=1}^{\infty} (n+1)(z+1)^n$.
- h) Find the length of the curve $y = \frac{4}{3} x^{3/2}$ for $0 \leq x \leq 20$.
- i) State and prove sufficient condition for a function to be analytic.
- j) Determine a, b, c, d so that function $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic.

Section - B

(4 × 5 = 20)

Q2) Expand $f(z) = \frac{1}{z^2(z-i)}$ as a Laurent's series about i and hence find the residue. There at.

Q3) Evaluate $\oint_C \frac{z-23}{z^2-4z-5} dz$, where C is the circle $|z-2| = 4$

Q4) Find the image of circle $|z-1| = 1$ in the w -plane under the mapping $w = z^2$.

Q5) Determine the analytic function whose real part is $e^x (\cos y - y \sin y)$.

Q6) Verify the Roll's theorem to the function $f(x) = e^{-x} \sin x$, $x \in [0, \pi]$.

Section - C

(2 × 10 = 20)

Q7) Evaluate $\int_{-\infty}^{\infty} \frac{\cos x}{x^2 + a^2} dx$.

Q8) solve $\nabla^2 u = 0$, under the condition ($h = k = 1$),

$$u(0, y) = 0, u(4, y) = 12 + y, \text{ for } 0 \leq y \leq 4;$$

$$u(x, 0) = 3x, u(x, 4) = x^2 \text{ for } 0 \leq x \leq 4.$$

Q9) A string of length ' ℓ ' is initially at rest in equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = b \sin^3 \frac{\pi x}{\ell}$. Find the displacement $y(x, t)$.

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