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B.Tech. (Sem. - 4th)

MATHEMATICS - III

SUBJECT CODE : CS - 204

Paper ID : [A0495]

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours **Maximum Marks : 60**

Instruction to Candidates:

1) Section - A is **Compulsory**.
2) Attempt any **Four** questions from Section - B.
3) Attempt any **Two** questions from Section - C.

Section - A

Q1) *(10 x 2 = 20)*

a) Define the Rolle's Theorem and Mean Value Theorem.

b) Find the poles from $\frac{e^z}{1+z^2}$.

c) How Taylor series is different from Laurent series?

d) Using Picard's method to find first approximation of $\frac{dy}{dx} = x + y^2, y(0) = 1$.

e) Explain Crank-Nicholson difference scheme for PDEs.

f) Find the inverse Laplace transform of $\frac{1}{(s-2)^2+3^2}$.

g) Define Translational transformation with example.

h) Name few methods used to solve boundary value problems.

i) Change $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dydx$, into polar co-ordinates.

j) Show that the function $f(x)=\sin\frac{1}{x}$ is continuous in $\left(0, \frac{2}{\pi}\right)$.

Section - B

(4 x 5 = 20)

Q2) Solve the equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by the method of separation of variables.

Q3) Use residue calculus to evaluate the integral $\int_0^{2\pi} \frac{d\theta}{5 - 4\sin\theta}$.

Q4) Prove that the function sin z is analytic and find its derivative.

Q5) Evaluate $\iint_R e^{2x+3y} dxdy$ over the triangle bounded by $x = 0, y = 0$ and $x + y = 1$.

Q6) Employ Taylor's method to obtain approximate of y at $x = 0.02$ for differential equation $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$.

Section - C

(2 x 10 = 20)

Q7) (a) Expand $f(z) = \frac{1}{z(z^2 - 3z + 2)}$ in a Laurent's series valid for the regions $|z| > 2$.

(b) Give an example of bounded function which is not Riemann integrable on [0, 1].

Q8) (a) Using Runge Kutta method of order 4, find $y(0.1)$ and $y(0.2)$ given that $\frac{dy}{dx} = xy + y^2, y(0) = 1$. (Take $h = 0.1$).

(b) If $A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} = 0$, then what are the conditions for this equation to be elliptic, parabolic and hyperbolic.

Q9) (a) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions $u(0, y) = u(1, y) = u(x, 0) = 0$ and $u(x, a) = \sin n\pi x$.

(b) Find all singularities of the function $f(z) = \frac{z - i}{(z^2 + 1)(z - 3)}$.

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