

Total No. of Questions: 09

**B.Tech. (2011 Onwards) (Sem. – 2)**  
**ENGINEERING MATHEMATICS – II**

**M Code: 54092**

**Subject Code: BTAM-102**

**Paper ID: [A1111]**

**Time: 3 Hrs.**

**Max. Marks: 60**

**INSTRUCTIONS TO CANDIDATES:**

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select at least TWO questions from each SECTION - B & C.

**SECTION A**

1. a) Find the general value of  $\log i^i$ .
- b) Solve the differential equation  $p = \sin(y - xp)$ , where  $p$  has its usual meaning.
- c) Test whether the differential equation  $(5x^3 + 12x^2 + 6y^2)dx + 6xydy = 0$  is exact or not? Give reasons. If not. Find its integrating factor which will make it exact.
- d) Show that  $e^z$  is a periodic function. Find its fundamental period.
- e) Test whether the set of vectors  $\{(1,0,0), (1,1,1), (1,2,3)\}$  is linearly independent or dependent.
- f) Examine the convergence / divergence of the series  $\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{1}{n}$
- g) Test the absolute convergence of the series  $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n\sqrt{n}}$
- h) If  $\lambda$  is an eigen value of a matrix  $A$  then prove that  $\lambda^{-1}$  is an eigen value of  $A^{-1}$ .
- i) For what values of 'k' the system of equation
$$x + y + z = 6; x + 2y + 3z = 10; x + 2y + kz = \lambda,$$
has unique solution.
- j) If  $x^2 - 2x \cos\theta + 1 = 0$ , then show that  $x^{2n} - 2x^n \cos n\theta + 1 = 0$

### SECTION B

2. a) Solve the differential equation:

$$y - 2px = \tan^{-1}(x p^2)$$

- b). Use method of variation of parameters to find the general solution of the differential equation:  $y'' - 2y' + y = e^x \log x$

3. a) Find the solution of the differential equation:

$$xy(1+xy^2)dy = dx$$

- b) Find the particular solution of the differential equation:

$$x^2 y'' + xy' + y = \log x \sin(\log x).$$

by using operator method.

4. a) Solve the simultaneous linear differential equation:

$$\frac{dx}{dt} + 2y + \sin t = 0, \quad \frac{dy}{dt} - 2x - \cos t = 0$$

- b) Find the particular solution of the differential equation:

$$y''' - 7y'' + 10y' = e^{2x} \sin x$$

5. An L-C-R circuit, the charge  $q$  on a plate of the condensor is given by the

equation:  $L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = E \sin pt$ , where  $\frac{dq}{dt} = i$  The circuit is tuned to

resonance so that  $p^2 = \frac{1}{LC}$ . If  $q = i = 0$  when  $t = 0$ , show that for small values of  $R/L$ , the current in the circuit at time  $t$  is given by  $(Et/2L) \sin pt$ .

### SECTION C

6. a) Find the eigen values and the corresponding eigen vectors of the matrix:

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

- b) Test the conditional convergence of the series.  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{n^2 + 1}$

7. a) Test the consistency of the system of equations:

$$x + 2y - z = 1; 3x - 2y + 2z = 2; 7x - 2y + 3z = 5,$$

and if consistent then solve it completely.

- b) Reduce the matrix  $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$  to normal form and hence find its rank.

8. a) Test for what values of 'x' for which the series

$$\frac{1}{1.2}x + \frac{1}{3.4}x^2 + \frac{1}{5.6}x^3 + \frac{1}{7.8}x^4 + \dots \infty$$

Converges / diverges.

- b) Examine the convergence / diverge of the series:

$$\frac{\sqrt{2}-1}{3^3-1} + \frac{\sqrt{3}-1}{4^3-1} + \frac{\sqrt{4}-1}{5^3-1} + \dots \infty$$

9. a) If  $\alpha, \beta, \gamma$  are the roots of the equation  $x^3 + px^2 + qx + p = 0$ , then prove that  $\tan^{-1} \alpha + \tan^{-1} \beta + \tan^{-1} \gamma = n\pi$  radians except in one particular case. Mention this case.

- b) If  $\sin^{-1}(u + iv) = \alpha + i\beta$ , then prove that  $\sin^2 \alpha$  and  $\cosh^2 \beta$  are the roots of the equation  $x^2 - x(1 + u^2 + v^2) + u^2 = 0$