Total No. of Questions: 09

B.Tech. (2011 Onwards) (Sem. – 1)

ENGINEERING MATHEMATICS – I

M Code: 54091

Subject Code: BTAM-101

Paper ID: [A1101]

Time: 3 Hrs. **Max. Marks: 60**

INSTRUCTIONS TO CANDIDATES:

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt FIVE questions from SECTION B & C carrying EIGHT marks each, selecting at least TWO questions each from SECTION - B & C.**
4. **Symbols used have their usual meanings. Statistical tables, if demanded, may be provided.**

SECTION A

1. a) Find asymptotes, parallel to axes, of the curve: $y = \frac{x^2+1}{x^2-1}$.

b) Write a formula to find the volume of the solid generated by the revolution, about x - axis, of the area bounded by the curve y = f(x), the x - axis and the ordinates x = a and x = b.

c) Find the value of $\frac{\partial (x,y)}{\partial (r,\theta)}$, where $x = r\cos\theta$ & $y = r\sin\theta$.

d) If an error of 1% is made in measuring the major and minor axes of an ellipse, what is the percentage error in its area?

e) Is the function $f(x,y,z) = \frac{4x^3+2y^2z}{x+2y+3z}$? If yes, what is its degree?

f) What is the value of $\iint xydxdy$ over the positive quadrant of the circle $x^2 + y^2 = 1$?

g) Give geometrical interpretation of $\int_1^2 \int_1^3 dxdy$

h) Show that for the vector field $\vec{F} = (x^2 - y^2 + x)\,\hat{\imath} - (2xy + y)\hat{\jmath}$, $\nabla \times \vec{F} = 0$.

i) Show that the vector field $\vec{F} = (-x^2 + yz)\,\hat{\imath} + (4y - z^2x)\,\hat{\jmath} + (2xz - 4z)\hat{k}$ is solenoidal.

j) State Green's theorem in plane.

SECTION B

2. Trace the following curves by giving their salient feature:

   a) $y^2(a-x) = x^2(a+x)$.

   b) $r = a(1-\cos\theta)$ (4,4)

3. a) Find the whole length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

   b) Use definite integral to find the area of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. (4,4)

4. a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 = -9(x+y+z)^{-2}$.

   b) State Euler's theorem for homogeneous functions and apply it to show that (4,4)

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u \quad , \qquad \text{where} \quad \sin u = \frac{x^2+y^2}{x+y}.$$

5. a) The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

   b) If $f(x,y) = \tan^{-1} xy$, compute $f(0.9, -1.2)$ approximately. (4,4)

SECTION C

6. a) Evaluate the following integral by changing the order of integration:

$$\int_0^1\int_0^{\sqrt{1-x^2}} y^2\,dx\,dy$$ (4,4)

   b) Evaluate the triple integral $\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$.

7. a) Find a unit vector normal to the surface $x^2+y^2+z^2= 9$ at the point (2, -1, 2).

   b) If $u = x^2+y^2+z^2$ & $\vec{v} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, Show that $\nabla.(u\vec{v}) = 5u$ (4,4)

8. a) If $\vec{F} = 3xy\hat{\imath} - y^2\hat{\jmath}$, evaluate, $\int_c \vec{F}.d\hat{R}$ where C is the curve in the xy-plane $y = 2x^2$ from (0,0) to (1,2).

   b) Compute $\int_c \vec{F}.\widehat{N}\,ds, Where\ \vec{F} = x\hat{\imath} + (z - zx)\hat{\jmath} - xy\hat{k}$ and S is the triangular surface with vertices (2, 0,0), (0, 2, 0) and (0, 0, 4). (4,4)

9. State Gauss Divergence theorem and verify it for $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ taken over the cube bounded by $x = 0, x = 1;\ y = 0, y = 1, z = 0, z = 1$.