Roll No. ....

Total No. of Questions : 09] **[Total No. of Pages : 02**

B.Tech. (Sem. – 3^{rd})

APPLIED MATHEMATICS - III

SUBJECT CODE : AM - 201

Paper ID : [A0303]

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours **Maximum Marks : 60**

Instruction to Candidates:

1) Section - A is **Compulsory**.
2) Attempt any **Four** questions from Section - B.
3) Attempt any **Two** questions from Section - C.

Section - A

Q1) ***(10 × 2 = 20)***

a) Define orthogonal functions.

b) State the Laplace transform of first and second derivatives.

c) Find the Laplace transform of sin 2t cos 3t.

d) State the sufficient condition for existence of Laplace transform of any function.

e) For $t^2\frac{d^2x}{dt^2} + t\frac{dx}{dt} + x = 0$; discuss the type of the point $t = 0$.

f) Define Error function and its one property.

g) Show that the function $\frac{\sinh z}{z^4}$ has a pole or order 3 at z = 0 with residue 1/6.

h) Form the partial differential equation by eliminating arbitrary constants a and b from the relation $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

i) Find all values of z for which $e^z = -2$.

j) State the CR – equations for a complex function.

Section - B

(4 × 5 = 20)

Q2) Evaluate the inverse Laplace transform of $\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$.

Q3) Find the Fourier series expansion of the function $f(x) = x^2; -2 \leq x \leq 2$.

Q4) Find the one Frobenius series solution of following equation about $x = 0$.

$$xy'' + y' + xy = 0.$$

Q5) Find the general integral of the following partial differential equation

$$\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^2 z \partial z}{\partial x^2 \partial y} + 4\frac{\partial z \partial^2 z}{\partial x \partial y^2} = 4\sin(2x + y).$$

Q6) Use generating function for Legendre polynomials to derive recursion formula

$$(n+1)P_{n+1}(x) = (2n+1)\,xP_n(x) - nP_{n-1}(x)$$

Section - C

(2 × 10 = 20)

Q7) Obtain the solution of the initial value problem using Laplace transform.

$$y'' + 4y' + 13y = e^{-t}, y(0) = 0, y'(0) = 2.$$

Q8) Use method of separation of variables to find the solution of heat conduction equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 \leq x \leq 1$, $t > 0$, with boundary conditions $u(0, t) = u(1, t) = 0$ and initial conditions $u(x, 0) = x(1 - x)$.

Q9) (a) Determine poles of the function $f(z) = \frac{1}{(e^z - 1)}$ and residue at each pole by expanding the function with Laurent's series. Hence evaluate

$$\int_c f(z)dz\ c;\ |z| = 1.$$

(b) Find the bilinear transformation which maps the points $z = -1, 0, 1$ in the z − plane onto the points $w = -i, 1, i$ in the w – plane.