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Total No. of Questions : 09]

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B.Tech. (Sem. - 4th)

MATHEMATICS - III

SUBJECT CODE : AM - 201 (2k3 Batch on Wards)

Paper ID : [A0865]

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours **Maximum Marks : 60**

Instruction to Candidates:

1) Section - A is **Compulsory**.
2) Attempt any **Four** questions from Section - B.
3) Attempt any **Two** questions from Section - C.

Section - A

Q1) *(10 × 2 = 20)*

a) State second shifting property and use it to find inverse laplace of $\frac{e^{-s}}{(s+1)^3}$.

b) Form the differential equation of all spheres whose centres lie on the z-axis.

c) State cauchy theorem and use it to evaluate $\int_C \frac{e^z}{z^2+1} dz$, where C is the curve $|z| = \frac{1}{2}$.

d) Obtain the value of b_n in the Fourier series expansion of the function

$$f(x) = \begin{cases} -(\pi + x), & -\pi < x < 0 \\ -(\pi - x), & 0 \le x < \pi \end{cases}.$$

e) Find the general solution of partial differential equation

$$4\frac{\partial^2 z}{\partial x^2} + 12\frac{\partial^2 z}{\partial x \partial y} + 9\frac{\partial^2 z}{\partial y^2} = 0.$$

f) Find the laplace transform of the function f(t) = [t], t > 0, where [ ] stands for greatest integer function.

g) Obtain the value of $J_{-1/2}(x)$.

h) Show that $P_n(-x) = (-1)^n P_n(x)$, where $P_n(x)$ is a legendre's polynomial of order n.

i) State the cartesian and polar form of Cauchy-Riemann equations.

j) Find the residue of $f(z) = \dfrac{ze^{iz}}{z^2+1}$ at each of its poles.

Section - B

(4 × 5 = 20)

Q2) Find the half range cosine series expansion of the function

$$f(x) = \begin{cases} x, & 0 \le x \le \dfrac{L}{2}. \\ L-x, & \dfrac{L}{2} \le x \le L \end{cases}$$

Q3) (a) Obtain the Laplace transform of a periodic function $f(t)$ of period T.

(b) Find the laplace transform of $\dfrac{\sin t}{t}$.

Q4) State and prove Rodrigue's formula.

Q5) Solve the partial differential equation :

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = \cos(y+2x).$$

Q6) For the function, f(z) defined by

$$f(z) = \begin{cases} \dfrac{(\bar{z})^2}{z}, & z \ne 0 \\ 0, & z = 0 \end{cases}$$

show that the C-R-equations are satisfied at (0, 0) but the function is not differentiable at (0, 0).

Section - C

(2 × 10 = 20)

Q7) (a) Solve the initial value problem

$y'' + 5y' + 5y = e^{-t}\sin t$, $y(0) = 0$,

$y'(0) = 1$, by using method of laplace transforms.

(b) Using Cauchy integral formula, evaluate $\int_C \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)(z-2)}\,dz$, where

$C : |z| = 3$.

Q8) (a) Using calculus of residues, evaluate the integral $I = \int_0^{2\pi} \frac{1}{a+\cos\theta}\,d\theta$, $a > 1$.

(b) Show that Legendre polynomials are orthogonal on the interval [–1, 1].

Q9) A bar 10cm long with insulated sides has its ends A and B maintained at temperature 50°C and 100°C respectively, until steady state condition prevails. The temperature at A is suddenly raised to 90°C and at the same time lowered to 60°C at B. Find the temperature distribution in the bar at time t.

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