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Total No. of Questions : 09]

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Paper ID [A0807]

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B.Tech. (Sem. - 3rd/4th)

APPLIED MATHEMATICS - III (AM - 201)

Time : 03 Hours

Maximum Marks : 60

Instruction to Candidates:

- 1) Section - A is **Compulsory**.
- 2) Attempt any **Four** questions from Section - B.
- 3) Attempt any **Two** questions from Section - C.

Section - A

Q1)

(10 × 2 = 20)

- a) Define the conditions on a function for its representation as Fourier series.
- b) Evaluate, using Laplace transform technique,

$$\int_0^{\infty} \frac{e^{-t} - e^{-3t}}{t} dt$$

- c) Find the inverse Laplace transform of $F(s) = \frac{1}{2} \log \left(\frac{s^2 + b^2}{s^2 + a^2} \right)$
- d) What are ordinary, regular singular and irregular singular points of an ordinary differential equation?
- e) What are orthogonal functions? State the orthogonality condition of Bessel functions.
- f) Form a partial differential equation by eliminating the arbitrary constants, a, b and c from $(x-a)^2 + (y-b)^2 + z^2 = c^2$.
- g) Form a differential equation by eliminating the arbitrary function $F(xy + z^2, x + y + z) = 0$
- h) Solve the differential equation $p + q = \sin x + \sin y$

- i) Find analytic function $f(z) = u(r, \theta) + i v(r, \theta)$ such that

$$v(r, \theta) = r^2 \cos 2\theta - r \cos \theta + 2.$$
- j) What are singularities of an analytic function. Find the nature and location
of singularities of $f(z) = (z+1) \sin \frac{1}{(z-2)}$

Section - B

(4 × 5 = 20)

Q2) Find the Fourier series of

$$f(x) = x^2 \quad 0 \leq x \leq \pi$$

$$= -x^2 \quad -\pi \leq x \leq 0$$

Q3) A beam has its ends damped at $x = 0$ and $x = l$. A concentrated load W acts vertically downward at the point $x = l/3$. Describe the governing boundary value problem and find the resulting deflection.

Q4) Using generating function for Legendre Polynomial $P_n(x)$, prove that

$$(n+1) P_{n+1}(x) = (2n+1)x P_n(x) - n P_{n-1}(x)$$

Q5) Solve the differential equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial z}{\partial x} \cdot \frac{\partial z}{\partial y} - 2 \frac{\partial^2 z}{\partial y^2} = (y-1)e^x$$

Q6) Discuss the transformation

$$w = \sqrt{z}.$$

Is it conformal at the origin.

Section - C

(2 × 10 = 20)

Q7) The currents I_1 and I_2 in mesh are given by differential eqs.

$$\frac{dI_1}{dt} - wI_2 = a \cos pt$$

$$\frac{dI_2}{dt} + wI_1 = a \sin pt$$

with $I_1 = I_2 = 0$ at $t = 0$. Using Laplace transformation technique, find the currents $I_1(t)$ & $I_2(t)$.

Q8) (a) Evaluate $\int_0^{2+i} (\bar{z})^2 dz$ along the real axis to 2 and then vertically to $2+i$.

(b) Using Charpit method, solve $2z + p^2 + qy + 2y^2 = 0$.

Q9) Determine the poles of the function

$$f(z) = \frac{z^2}{(z-1)^2(z+2)}$$

and residue at each pole.

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