

Total No. of Questions : 09]

[Total No. of Pages : 02

Paper ID [AM201]

(Please fill this Paper ID in OMR Sheet)

B.Tech. (Sem. - 3rd/4th)

MATHEMATICS - III (AM - 201)

Time : 03 Hours

Maximum Marks : 60

Instruction to Candidates:

- 1) Section - A is Compulsory.
- 2) Attempt any Four questions from Section - B.
- 3) Attempt any Two questions from Section - C.

Section - A

Q1)

(10 × 2 = 20)

- a) Can $f(x) = \tan x$ be expanded as a Fourier series in the interval $(-\pi, \pi)$.
- b) Give the sufficient condition for the existence of Laplace transform of $f(t)$.
- c) If $L\{f(t)\} = \frac{1}{s} e^{-\frac{1}{s}}$, find the Laplace transform of $e^{-t} f(3t)$.
- d) Find the $L^{-1} \left\{ \frac{e^{-3s}}{s^2} \right\}$.
- e) Write algorithm of power series method for solution of differential equations.
- f) For the Bessel's function prove the recurrence relation

$$\frac{d}{dx} \{x^p J_p(x)\} = x^p J_{p-1}(x).$$

- g) Form the partial differential equation from $F(xy + z^2, x + y + z) = 0$.
- h) Is the function $u(x, y) = 2xy + 3xy^2 - 2y^3$, a harmonic function?
- i) Give the definition of Conformal transformation.
- j) Find the poles and singularity from $\frac{e^z}{1 + z^2}$.

Section - B

(4 × 5 = 20)

Q2) Solve the system of differential equations using Laplace transform

$$2 \frac{dx}{dt} + \frac{dy}{dt} - x - y = e^{-t} \text{ and } \frac{dx}{dt} + \frac{dy}{dt} + 2x + y = e^t.$$

Given that $x(0) = 2, y(0) = 1$.

Q3) Using convolution theorem find the inverse of $\frac{1}{s^2(s^2 + 1)}$.

Q4) Find the series solution of differential equation $(1 - x^2) y'' - 2x y' + 6y = 0$.

Q5) Solve $(D_x^4 - D_x^3 D_y + 2D_x^2 D_y^2 - 5D_x D_y^3 + 3D_y^4)z = 0$.

Q6) Evaluate $\oint_C |z|^2 dz$, around the square with vertices at $(0,0), (1,0), (1,1)$ and $(0,1)$.

Section - C

(2 × 10 = 20)

Q7) Find the Fourier series expansion of $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \leq x \leq \pi \\ 1 - \frac{2x}{\pi}, & 0 \leq x \leq \pi \end{cases}$

Also deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

Q8) Solve Laplace's equation in rectangle with $u(0, y) = 0, u(a, y) = 0, u(x, b) = 0$ and $u(x, 0) = f(x)$.

Q9) Find the Laurent series of $f(z) = \frac{1}{(1-z)(z-2)}$ for the following intervals

(a) $1 < |z| < 2$

(b) $|z| > 2$

