Roll No. **Total No. of Pages : 03**

Total No. of Questions : 09

B.Tech. (EE / EEE / EIE / ECE / BME-2007 & Onward Batches) (Sem.–3rd)

ENGG. MATHEMATICS / APPLIED MATHEMATICS-III

Subject Code : AM-201

Paper ID : [A0303]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

1. Answer briefly :

(a) If a function $f(z) = u + iv$ is analytic in a domain D, then prove that its component functions u and v are harmonic in D.

(b) Explain what do you understand by conformal mapping.

(c) Find the residuce of $f(z) = \dfrac{e^{z^2}}{(z-i)^3}$ at its poles.

(d) Define unit impulse function and hence find its Laplace transform.

(e) Find the Laplace transform of $f(t) = \sin^3 2t$.

(f) Prove that $L\left\{\int_0^t f(u)\,du\right\} = \dfrac{F(s)}{S}$, where $F(S) = L\{f(t)\}$.

(g) Obtain the complete solution of the equation

$$2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0.$$

(h) Form the partial differential equation by eliminating the arbitrary function from

$$f(x^2 + y^2, z - xy) = 0$$

(i) Show that $J_{\frac{-1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

(j) Find the Fourier coefficient 'a_0' of $f(x) = | \sin x |$ in the interval $(-\pi , \pi)$.

SECTION-B

2. Obtain the Fourier series expansion of $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ and hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$$ (5)

3. (a) Define unit step function and hence find its Laplace transform.

(b) Find the inverse Laplace of $\cot^{-1}(S)$. (3, 2)

4. (a) Solve the linear equation :

$$x^2(y - z)\, p + y^2\, (z - x)\, q = z^2\, (x - y)$$

Where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

(b) Express $5x^3 + x$ in terms of Legendre polynomials. (3, 2)

5. Show that the function :

$$f(z) = \begin{cases} \dfrac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}, & z \neq 0 \\ 0 & , z = 0 \end{cases}$$

satisfies Cauchy-Riemann equations at the origin yet $f'(0)$ dose not existo (5)

6. (a) Evaluate $\int_0^{1+i} (x - y + ix^2)dz$ along the straight line from $z = 0$ to $z = 1 + i$.

(b) Find the complete solution of

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \cos x \cos 2y \qquad (2,3)$$

SECTION-C

7. (a) Obtain the half range sine series for the function

$$f(x) = \begin{cases} x, 0 < x < \frac{\pi}{2} \\ \pi - x, \frac{\pi}{2} < x < \pi \end{cases}$$

(b) Solve : $\frac{d^2x}{dt^2} + 9x = \cos 2t$, $x(0) = 1$, $x\left(\frac{\pi}{2}\right) = -1$ by using method of Laplace Transform. (4, 6)

8. (a) State and prove the orthogonality property of Bessel functions.

(b) State Cauchy integral formula and use it to evaluate :

$$\int_C \frac{\sin z}{\left(z - \frac{\pi}{4}\right)^3} dz$$

where C is $\left|z - \frac{\pi}{4}\right| = \frac{1}{2}$. (6, 4)

9. (a) A bar 100 cm long, with insulated sides, has its ends kept at 0°C and 100°C until steady state conditions prevail. If B is suddenly reduced to 0°C and maintained at 0°C, find the temperature at a distance x from A at time t.

(b) Construct an analytic function whose real part is $u(x, y) = 2xy + 2x$. (6, 4)