

Roll No.....

Total No.of Pages:02

MAY –2014
MATHEMATICS I
Paper Code (BSBC-103)
Paper Id. [B1110]

Time Allowed: 3 Hrs

Max. Marks: 60

NOTE: Section A is compulsory. Attempt any four questions from Section B.

SECTION A

Q1.

- a) Prove that for any two sets A and B , $(A \cup B)^c = A^c \cap B^c$.
b) Using principle of mathematical induction, prove that

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

- c) Solve the recurrence relation

$$S(n) - 8S(k-1) + 12S(k-2) = 0, S(0) = 54, S(1) = 308$$

- d) Define chromatic number of a graph.
e) Differentiate the Eulerian path from the Hamiltonian path.
f) Give an example of a relation which is both symmetric and anti-symmetric.

- g) Prove that $\sim(p \leftrightarrow q) = \sim p \leftrightarrow q = p \leftrightarrow \sim q$.

- h) If $M_r = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $M_s = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ are the relational matrices of the relation r and s on a set $A = \{1, 2, 3, 4\}$, find $s \circ r$.

- i) Write any two properties of a binary tree.
j) Draw the digraph of the relation R defined by xRy if x divides y in the set $A = \{2, 3, 4, 6, 8, 9, 12\}$.

Section-B

Q2. (a) Let R be a relation from the set A to set B . and S be a relation from set B to set A , then

$$(S \circ R)^{-1} = R^{-1} \circ S^{-1}$$

(b) If R_1 and R_2 are transitive relations on a set A then prove that $R_1 \circ R_2$ is also a transitive relation.

Q3. (a) Solve the recurrence relation

$$S(n) - 4S(n-1) + 3S(n-2) = n^2$$

(b) Let $A = \{a, b, c, d\}$. Let r be the relation on A with adjacency matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}.$$

Check whether r is a partial order or an equivalence relation on A .

Q4. (a) Prove by mathematical induction

$$\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$$

(b) Are the following arguments valid? If valid, construct a format proof, if not explain with the reason.

Wages will increase if and only if there is an inflation. If there is an inflation, then cost of living will increase. Wages will increase, therefore, the cost of living will increase.

Q5. (a) Determine whether the function on the set of rational numbers defined by xRy if $x - y$ is an integer is an equivalence relation.

(b) Partition the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ into the minsets generated by $B_1 = \{5, 6, 7\}$, $B_2 = \{2, 4, 5, 9\}$, $B_3 = \{3, 4, 5, 6, 8, 9\}$.

Q6. (a) Show that every square matrix can be written as a sum of symmetric and skew-symmetric matrix.

(b) Find the inverse of the matrix

$$\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Q7. (a) Prove that the chromatic number of graph C_n where C_n is the cycle with ' n ' vertices is either 2 or 3.

(b) State and prove Euler's theorem on graphs.

4 × 10 = 40 marks

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