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Total No. of Questions : 07

B.C.A. (Sem.–1)
MATHEMATICS-I
Subject Code : BSBC-103 (2011 Batch)
Paper ID : [B1110]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTION TO CANDIDATES :

1. **SECTION-A is COMPULSORY.**
2. **Attempt any FOUR questions from SECTION-B.**

SECTION-A (10 × 2 = 20 Marks)

1. (a) State the contrapositive and converse of the following implication :
"*I will not take the examination if I go for a movie or I have headache.*"

(b) Which of the following sentences are propositions and what are their truth values ?
(i) $4 + x = 3 + y$
(ii) $x + y = y + x$ for every pair of real numbers x and y.

(c) Let the universe of discourse of x consist of all real numbers. Determine the truth value of the statement $\forall x\ (x^2 = x)$.

(d) List all the elements of the set
$A = \{x \mid x \text{ is a square of an integer and } x < 100\}$

(e) Prove that for any two sets A and B
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ where for any set X, $\overline{X}$ denotes the compliment of X.

(f) Let A be the set $\{1, 2, 3, 4\}$. List all ordered pairs which belong to the relation $R = \{(a, b) \mid a \text{ divides } b\}$

(g) Find the first five terms of the sequence defined by the recurrence relation $a_n = a_{n-1} + 3a_{n-2}$, $a_0 = 1$, $a_1 = 2$.

(h) Find the term independent of x in the expansion of $\left(2x + \frac{1}{x^2}\right)^9$.

(i) How many edges are there in a graph with ten vertices each of degree six.

(j) Define a complete graph K_n and *a* cycle C_n on *n* vertices.

SECTION-B **(4 × 10 = 40 Marks)**

2. (a) Let R be a relation from A = $\{a_1, a_2 \ldots, a_m\}$ to B = $\{b_1, b_2, \ldots, b_n\}$. What is the matrix representation of R ?

(b) Suppose that the relations R_1 and R_2 on a set A are represented by the matrices.

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$ (2,8)

What are the matrices representing $R_1 \cup R_2$ and $R_1 \cap R_2$?

3. (a) Show that $\sim (p \vee (\sim p \wedge q))$ and $\sim p \wedge \sim q$ are logically equivalent.

(b) Prove that $A \subseteq B$ is a necessary and sufficient condition for $A \cup B = B$. (5,5)

4. (a) Is the following argument valid ? Justify your answer.

"If taxes are lowered, then income rises".

Income rises :

∴ Taxes are lowered.

(b) Prove, by using Mathematical induction, that for every positive number *n*, the number $2^{2n} - 1$ is divisible by 3. (5,5)

5. (a) Solve the recurrence relation :

$$a_n = 4a_{n-1} + 5a_{n-2}, \quad a_1 = 2, a_2 = 6$$

(b) Find the two middle terms in the expansion of the binomial $\left(3x^2 + \dfrac{5}{y^2}\right)^{11}$.

(5,5)

6. (a) Let a graph G have more than two vertices of odd degree. Prove that there can be no Euler path in G.

(b) What is the chromatic number of K_n, the complete graph on *n* vertices? Justify your answer.

7. Prove that a tree with *n* vertices has *n* – 1 edges.