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Total No. of Pages : 03

Total No. of Questions : 09

**B.Tech.(CSE) / (IT) (2011 Batch) (Sem.-4)**

**MATHEMATICS-III**

**Subject Code : BTCS-402**

**Paper ID : [A1184]**

**Time : 3 Hrs.**

**Max. Marks : 60**

**INSTRUCTION TO CANDIDATES :**

1. **SECTION-A** is **COMPULSORY** consisting of **TEN** questions carrying **TWO** marks each.
2. **SECTION-B** contains **FIVE** questions carrying **FIVE** marks each and students has to attempt any **FOUR** questions.
3. **SECTION-C** contains **THREE** questions carrying **TEN** marks each and students has to attempt any **TWO** questions.

**SECTION-A**

**1. Write briefly :**

- (a) Find the Fourier cosine series for the function  $f(x) = 1, 0 \leq x \leq 2$ .
- (b) Define Laplace transformation. State sufficient conditions for the existence of Laplace transformation of any function  $f(t)$ .

(c) Given show that  $L\left(2\sqrt{\frac{t}{\pi}}\right) = \frac{1}{s^{\frac{3}{2}}}$ , show that  $L\left(\sqrt{\frac{1}{\pi t}}\right) = \frac{1}{\sqrt{s}}$ .

(d) Form the partial differential equation from  $f(x^2 + y^2, z - xy) = 0$ .

(e) Solve :  $2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0$

(f) Using C-R equations prove that  $f(z) = |z|^2$  is not analytic at any point.

(g) Explain the difference between direct and iterative methods for solving simultaneous linear equations. Mention the different available methods for solving them.

- (h) Explain geometrically the difference between Euler's and modified Euler's method.
- (i) Write down the different properties of the normal distribution.
- (j) Write down any three properties of t-distribution.

### SECTION-B

2. Expand the function  $f(x) = x \sin x$  as a Fourier series in the interval  $[-\pi, \pi]$ .

Also deduce that :  $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \dots = \frac{1}{4}(\pi - 2)$

3. Find the Laplace transforms of (i)  $f(t) = \frac{\cos at - \cos bt}{t}$ , (ii)  $f(t) = te^{-at} \cos 3t$
4. Find the largest eigen-value and the corresponding eigen-vector for the equations:  
 $(2 - \lambda)x_1 - x_2 = 0$ ,  $-x_1 + (2 - \lambda)x_2 - x_3 = 0$ ,  $-x_2 + (2 - \lambda)x_3 = 0$  by Rayleigh power method.
5. Using Runge-Kutta method of fourth-order, solve :

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2} \text{ with } y(0) = 1 \text{ at } x = 0.2, 0.4 .$$

6. The probability that a pen manufactured by a company will be defective is  $\frac{1}{10}$ .  
If 2 such pens are manufactured, find the probability that (i) exactly two will be defected, (ii) at least two will be defective, (iii) none will be defective.

### SECTION-C

7. (a) Show that the function  $v(x, y) = \ln(x^2 + y^2) + x - 2y$  is harmonic. Find its conjugate harmonic function  $u(x, y)$  and the corresponding analytic function  $f(z)$ .
- (b) Find the general solution of the partial differential equation :

$$px(z - 2y^2)p = (z - qy)(z^2 - y^2 - 2x^3).$$

8. (a) If  $u = \log \tan \left( \frac{\pi}{4} + \frac{\theta}{2} \right)$ , prove that (i)  $\tan h \frac{u}{2} = \tan \frac{\theta}{2}$ , (ii)  $\theta = -i \log \tan$

$$\left( \frac{\pi}{4} + i \frac{u}{2} \right).$$

- (b) Solve the system of equations :  $5x - 2y + z = 4$ ,  $7x + y - 5z = 8$ ,  $3x + 7y + 4z = 10$   
using Gauss elimination method with partial pivoting.

9. The theory predicts the proportion of beans in four groups,  $G_1, G_2, G_3, G_4$  should be in the ratio 9:3:3:1. In an experiment with 1600 beans, the number in the four groups were 882, 313, 287 and 118. Does the experiment result support the theory?