Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 02**

Total No. of Questions : 09

B.Tech. (BME/ECE/EE/EEE/EIE) (Sem.–3)

ENGG. MATHEMATICS / APPLIED MATHEMATICS – III

Subject Code : AM-201

Paper ID : [A0303]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

1. Write briefly :

1. Find the Fourier series expansion of the periodic function

   $f(x)=x,\ -\pi\leq x\leq\pi,\ f(x+2\pi)=f(x).$

2. Write the Euler's formula for Fourier coefficients.
3. If $L[f(t)] = F(s)$, show that $L[f(at)] = (1/a)F(s/a)$, where $L[f(t)]$ represents the Laplace transform of the function $f(t)$.
4. Find the Laplace transform of (i) $e^{5t}t^2$ (ii) $t \sin 4t$.
5. Show that the function $u(x, y) = y^3 - 3x^2y$ is harmonic.
6. Find the general and principal value of i^i.
7. Define Bessel's differential equation and Bessel function of first kind.
8. Eliminate the arbitrary function from $z = f(x^2 + y^2)$ to obtain a first order partial differential equation.
9. Write one dimensional wave equation and heat equation.
10. Find the value of Legendre polynomial $P_4(x)$.

SECTION-B

2. Find a Fourier series to represent a function $f(x) = x - x^2$ from $x = -\pi$ to $x = \pi$ and hence show that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ... = \frac{\pi^2}{12}$.

3. Find the solution of the initial value problem, using Laplace transforms
$ty'' + 2ty' + 2y = 2,\ y(0) = 1,\ y'(0)$ is arbitrary.

4. Find all possible Laurent series expansions of the function $f(z) = \frac{1}{(z+1)(z+2)^2}$ about the point $z = 1$.

5. Show that $\left[x^v J_v(x)\right]' = x^v J_{v-1}(x)$, where $J_v(x)$ is the Bessel function of first kind.

6. Solve the differential equation $r + 2s + t = 2 \sin y - x \cos y$.

SECTION-C

7. (a) Use residue theorem to evaluate the integral $\oint_C \frac{1}{z^4+1} ds; C : |z-1| = 1.$ (6)

   (b) Show that the function $u(x, y) = 2x + y^3 - 3x^2 y$ is harmonic. Find its harmonic conjugate and hence construct the corresponding analytic function $f(z)$. (4)

8. (a) Find the general solution of Lagrange's equation
$px\ (x + y) = qy\ (x + y) - (x - y)\ (2x + 2y + z)$ (5)

   (b) Let $f(t)$ be piecewise continuous on $[0, \infty]$, be of exponential order and periodic with period T. Then $L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t) dt, s > 0$ (5)

9. (a) Find the Fourier cosine series of the function $f(x) = \begin{cases} x^2 & 0 \le x \le 2, \\ 4 & 2 \le x \le 4 \end{cases}$ (5)

   (b) Find the inverse Laplace transform of the functions (i) $\frac{e^{-s}}{s^4}$ and (ii) $\frac{1}{s(s^2+a^2)}$. (5)