Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 02**

Total No. of Questions : 09

B.Tech. (2011 Onwards) (Sem.–1)

ENGINEERING MATHEMATICS – I

Subject Code : BTAM-101

Paper ID : [A1101]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

l. Solve the following :

a) Find the length of any arc of the curve $r = a\sin^2\dfrac{\theta}{2}$.

b) If $z = f(x, y)$ and $x = e^u + e^{-v}$, $y = e^{-u}e^v$, prove that $\dfrac{\partial z}{\partial u} - \dfrac{\partial z}{\partial v} = x\dfrac{\partial z}{\partial x} - y\dfrac{\partial z}{\partial y}$.

c) In polar co-ordinates $x = r\cos\theta$, $y = r\sin\theta$, show that $\dfrac{\partial(x, y)}{\partial(r, \theta)} = r$.

d) Using Euler's theorem, prove that if $\tan u = \dfrac{x^3 + y^3}{x - y}$, then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$.

e) Write Taylor's series for a function of two variables.

f) Find the value of 'a' for which the vectors $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} + a\hat{j} + 3\hat{k}$ are perpendicular.

g) If $r = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\vec{r}| = r$, show that $\nabla f(r) = f'(r)\,\nabla r$.

h) State Stoke's theorem.

i) Find the volume common to the two cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.

j) Evaluate $\int_C (x^2 + yz)\,dS$, where C is the curve defined by $x = 4y$, $z = 3$ from $\left(2, \dfrac{1}{2}, 3\right)$ to $(4, 1, 3)$.

SECTION-B

2. Find the radius of curvature at any point of following curves :

   a) $x = a(\cos t + t \sin t),\ y = a(\sin t - t \cos t)$ (4)

   b) $S = a \log(\sec\psi + \tan\psi) + a \sec\psi \tan\psi$ (4)

3. The cardiod $r = a(1 + \cos\theta)$ revolves about the initial line. Find the volume of the solid generated. (8)

4. a) Find the minimum value $x^2 + y^2 + z^2$ of subject to the condition that $xyz = a^3$. (4)

   b) Find the maximum and minimum values of $2(x^2 - y^2) - x^4 + y^4$. (4)

5. a) If $f(x, y) = \tan^{-1}(xy)$, find an approximate value of $f(1.1, 0.8)$ using the Taylor's series linear approximation. (3)

   b) Show that the function $f(x,y) = \begin{cases} \dfrac{x^3 + 2y^3}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$

   (*i*) is continuous at (0, 0) (*ii*) possesses partial derivatives at (0, 0) . (5)

SECTION-C

6. Find the centre of gravity of a plate whose density $\rho(x, y)$ is constant and is bounded by the curves $y = x^2$ and $y = x + 2$. Also, find the moment of inertia about the axis. (8)

7. a) If $a = \sin\theta\hat{i} + \cos\theta\hat{j} + \theta\hat{k}, b = \cos\theta\hat{i} - \sin\theta\hat{j} - 3\hat{k}$ *and* $c = 2\hat{i} + 3\hat{j} - \hat{k}$,

   find $\dfrac{d}{d\theta}(\vec{a} \times (\vec{b} \times \vec{c}))$ at $\theta = 0$. (4)

   b) A particle moves along the curve $x = 3t^2$, $y = t^2 - 2t$ and $z = t^3$. Find its velocity and acceleration at $t = 1$ in the direction of $\hat{i} + \hat{j} + \hat{k}$. (4)

8. Verify Gauss divergence theorem for $\vec{f} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$, taken over the region bounded by the cylinder $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. (8)

9. a) Evaluate the integral $\displaystyle\int_0^2 \int_0^{\frac{y^2}{2}} \frac{y}{\sqrt{x^2 + y^2 + 1}}\, dxdy$. (4)

   b) Prove that $div(f\vec{v}) = f(div\vec{v}) + (grad f).\ \vec{v}$, where f is scalar function. (4)