

SECTION-B

2. Obtain the Fourier series for the function $f(x)$ given by

$$f(x) = 1 + \frac{2x}{\pi}, \quad -\pi \leq x \leq 0$$

$$= 1 - \frac{2x}{\pi}, \quad 0 \leq x \leq \pi$$

Deduce that, $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$.

3. Find : $L^{-1} \left\{ \log \frac{s+1}{s-1} \right\}$.

4. With usual notations, prove that $\frac{d}{dx} [x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$.

5. Solve : $z^2(p^2 + q^2) = x^2 + y^2$.

6. Find an analytic function $f(z) = u + iv$ given that,

$$u + v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$$

SECTION-C

7. Solve : $\frac{d^2 y}{dt^2} + 9x = \cos 2t$, if $x(0) = 1$, $x\left(\frac{\pi}{2}\right) = -1$.

8. A tightly stretched string of length l with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $v_0 \sin^3 \frac{\pi x}{l}$. Find the displacement $y(x, t)$.

9. Prove that $\int_0^\pi \frac{a d\theta}{a^2 + \sin^2 \theta} = \frac{\pi}{\sqrt{1+a^2}}$, $a > 0$.