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Total No. of Questions : 09

B.Tech.(CSE/IT) (Sem.–4)
MATHEMATICS – III
Subject Code : CS-204
Paper ID : [A0495]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTION TO CANDIDATES :
1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

1. **Write briefly :**

(a) Show that the function e^z is analytic in finite z-plane.

(b) Find general and principal values of $\left(1+\sqrt{3i}\right)^{1+i}$.

(c) Find the residues at all singular points of $\dfrac{z^2}{\left(z^2+1\right)^2}$.

(d) State Rolle's Theorem.

(e) Using Lagrange's Mean Value Theorem, show that : $|\cos b - \cos a| \leq |b - a|$.

(f) Obtain approximate value of $y(1.2)$ for the initial value problem $y' = -2xy^2$, $y(1) = 1$, using Taylor's series method with step size $\lambda = 0.1$

(g) Obtain Picard's second approximate solution of the initial value problem

$$\frac{dy}{dx} = \frac{x^2}{y^2+1}\ ,\ y(0)= 0.$$

(h) Write Milne's predictor-corrector formulas.

(i) Find general and principal values of $\log(-1)$.

(j) Define conformal mapping.

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SECTION-B

2. Find the volume of the solid in the first octant bounded by the parabolic $z = 36 - 4x^2 - 9y^2$.

3. Find the centre of gravity of a plate whose density $p(x, y)$ is constant and is bounded by the curves $y = x^2$ and $y = x + 2$. Also find the moment of inertia about the axis.

4. If $f(z)\, u + iv$ is an analytic function of $z = x + iy$ and $u - v = e^{-x}[(x - y)\sin y - (x + y)\cos y]$ then find u and v.

5. Using Runge-Kutta method find $y(1.2)$ for the initial value problem $\dfrac{dy}{dx} = x^2 + y^2$, $y(1) = 1.5$, by taking step size $\lambda = 0.1$.

6. Solve the partial differential equation $\dfrac{\partial^2 z}{\partial x^2} - 2\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = 0$ by the method of separation of variables.

SECTION-C

7. (i) Obtain the terms upto z^3 in the Taylor Series expansion of $f(z) = \dfrac{z^2 + \sin^2 z}{1 - \cos z}$.   6

   (ii) Determine the angle of rotation at the point $z = \dfrac{1+i}{2}$ under the mapping $w = z^2$. Also find its scale factor.   4

8. (i) Evaluate the integral $\displaystyle\int_{-\infty}^{\infty} \dfrac{\sin^2 2x}{1 + x^2}\, dx$.   5

   (ii) Using Laplace transforms. Find the solution of the initial value problem $\dfrac{\partial u}{\partial x} + x\dfrac{\partial u}{\partial t} = xt$, $u(x, 0) = 0$, $u(0, t) = t$.   5

9. An insulated rod of length l has its ends A and B maintained at 0ºC and 100ºC respectively until steady state conditions prevail. If B is suddenly reduced to 0ºC and maintained at 0ºC, find the temperature at a distance x from A at time t. Also find the temperature if the change consists of raising the temperature of A to 20ºC and reducing that of B to 80ºC.   10