

Roll No.

Total No. of Pages : 02

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B.Tech.(BME/ECE/EE/EEE/EIE/Textile) (Sem.-3)

**APPLIED MATHEMATICS – III /
ENGINEERING MATHEMATICS**

Subject Code : AM-201

Paper ID : [A0303]

Time : 3 Hrs.

Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY** consisting of **TEN** questions carrying **TWO** marks each.
2. **SECTION-B** contains **FIVE** questions carrying **FIVE** marks each and students have to attempt any **FOUR** questions.
3. **SECTION-C** contains **THREE** questions carrying **TEN** marks each and students have to attempt any **TWO** questions.

SECTION-A

1. Write briefly :

1. State Euler's formula for the coefficients a_o , a_n and b_n in the fourier series expansion of a function $f(x)$ as $\frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi + \sum_{n=1}^{\infty} b_n \sin n\pi$ in the internal $(d, d + 2\pi)$.
2. Express $4x^3 - 2x^2 - 3x + 8$ in terms of Legendre polynomials.
3. Derive a partial differential equation by eliminating the arbitrary constants from the equation $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$
4. Solve the partial differential equation $(x^2 - y^2 - z^2)p + 2xyq = 2xz$
5. State Cauchy Riemann equations in Cartesian and polar coordinates.
6. Let $f(z) = u + iv$ be an analytic function of z , prove that $u(x, y)$ and $v(x, y)$ both satisfy the Laplace equation.
7. Find the Laplace transform of $\cos^2 2t$.
8. State the change of scale property of Laplace transform.
9. Define the Legendre's equation of order n . What are its particular solutions called?
10. What is the coefficient of $\sin nx$ in the Fourier series representation of $x - x^2$ from $x = -\pi$ to $x = \pi$.

SECTION-B

2. Prove that $J_n''(x) = \frac{1}{4} [J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)]$
3. Solve the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = \cos(2x+y)$$
4. Determine the analytic function whose real part is $\frac{y}{x^2 + y^2}$.
5. Find the inverse Laplace transform of $\frac{s}{s^4 + 4a^4}$.
6. Solve the differential equation $(D^3 - 3D^2 + 3D - 1)y = t^2 e^t$ given that $y(0) = 1, y'(0) = 0, y''(0) = -2$; using Laplace transforms.

SECTION-C

7. Obtain Fourier series for the function $f(x)$ given by $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}, & 0 \leq x \leq \pi \end{cases}$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$.

8. The diameter of a semi circular plate of radius a is kept at 0°C and the temperature at the semi-circular bounding is $T^\circ\text{C}$. Show that the steady state temperature in the plate is given by $u(r, \theta) = \frac{4T}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \left(\frac{r}{a}\right)^{2n-1} \sin(2n-1)\theta$
9. Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4 \cos \theta} d\theta$, by using complex integration.