Roll No. | | | | | | | | | | | | **Total No. of Pages : 02**

Total No. of Questions : 09

B.Tech.(CSE/IT) (Sem.–4)

MATHEMATICS – III

Subject Code : CS-204

Paper ID : [A0495]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTION TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

1. Write briefly :

a) State Rolle's theorem.

b) Using the concept of multiple integrals, find the area common to the parabolas, $y^2 = 4x$ and $x^2 = 4y$.

c) Find the value of z where the function, $w = \log z$ is not analytic.

d) State Laurent's theorem.

e) For the function, $f(z) = \dfrac{z^2 - 3}{(z-3)^3 (z+2)^2}$, find the residue at the "pole of order 2".

f) Define the term "conformal mapping".

g) State Convolution theorem for Fourier transforms.

h) Write down the three possible solutions for the Laplace equation, $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$.

i) Explain briefly the Picard's method for the numerical solution of the differential equation, $\dfrac{dy}{dx} = f(x, y)$.

j) Write the general linear partial differential equation of second order in two independent variables. Under what condition, it is (i) elliptic (ii) parabolic (iii) hyperbolic?

SECTION-B

2. Evaluate $\iiint \frac{dx\,dy\,dz}{(x+y+z+1)^3}$ over the tetrahedron bounded by the co-ordinate planes and the plane, $x + y + z = 1$.

3. State and prove the Cauchy's theorem.

4. Find the bilinear transformation which maps the points 1, i , –1 of the z-plane onto i, 0, $-i$ of the w-plane respectively.

5. Apply Runge- Kutta method of order 4 , to find an approximate value of y for $x = 0.1$, if $\frac{dy}{dx} = x^2 - y$, given that $y = 1$ when $x = 0$.

6. Employ Fourier transform method to solve the boundary value problem,

   $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}, t > 0, 0 < x < \infty,$ subject to the conditions :

   (i) $u(0,t) = u_0$, a constant

   (ii) $u(x,0) = 0$

SECTION-C

7. A string of length l is initially at rest in equilibrium position and each of its points is given the initial velocity, $\frac{\partial y}{\partial t} = b \sin^3\left(\frac{\pi x}{l}\right)$. Find the displacement $y(x,t)$.

8. By contour integration, evaluate $\int_0^{2\pi} \frac{d\theta}{5 - 3\cos\theta}$.

9. Solve the equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square with sides $x = 0 = y$, $x = 3 = y$ with $u = 0$ on the boundary and mesh length = 1.