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B.Tech. (Sem.-2)

ENGINEERING MATHEMATICS-II

Subject Code : BTAM-102 (2011 Batch)

Paper ID : [A1111]

Time : 3 Hrs.

Max. Marks : 60

INSTRUCTION TO CANDIDATES :

1. SECTION-A is COMPULSORY.
2. Attempt any FIVE questions SECTION - B & C.
3. Select at least TWO questions from SECTION - B & C.

SECTION-A

(10 × 2 = 20 Marks)

1. (a) Find the general value of  $\log(-1 + \sqrt{3}i)$ .

(b) Solve the differential equation  $\frac{d^3y}{dx^3} + y = 0$

(c) Under what conditions on 'a, b, c and d', the differential equation  $(a \sinh x \cos y + b \cosh x \sin y) dx + (c \sinh x \cos y + d \cosh x \sin y) dy = 0$ , is exact ?

(d) Find all the roots of  $(-8i)^{1/3}$ .

(e) Test for what values of 'k' the set of vectors  $\{(k, 1, 1), (0, 1, 1), (k, 0, k)\}$  is linearly independent.

(f) Examine the convergence / divergence of the series  $\sum_{n=1}^{\infty} \frac{x^n}{(2n)!}$

(g) Test the absolute convergence of the series  $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\log n)^2}$

(h) Test whether the matrix  $\begin{pmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}$  is diagonalisable or not ?

- (i) For what values of 'k', the system of equation  
 $x + y + z = 2$  ;  $x + 2y + z = -2$ ;  $x + y + (k - 5) z = k$ ,  
 has no solution.
- (j) Express  $\cos 6\theta$  in terms of powers of  $\cos\theta$  .

### SECTION-B

(4 × 5 = 20 Marks)

2. (a) Find the general solution of the differential

$$(3x^2y^3 e^y + y^3 + y^2) dx + (x^2y^3 e^y - xy) dy = 0$$

- (b) Obtain the general solution of the equation  $y'' + 3y' + 2y = \sin(e^x)$ ,  
 by using method of variation of parameters .

3. (a) Solve the following simultaneous differential equation

$$\frac{dx}{dt} + y = \sin t, \quad \frac{dy}{dt} + x = \cos t, \quad y(0) = 0, \quad x(0) = 2$$

- (b) Find the complete solution of the differential equation

$$(x + 1)^2 y'' + (x + 1)y' + y = \sin(2 \log(1 + x)).$$

by using operator method.

4. (a) Solve the differential equation  $x(\frac{dy}{dx} + y) = 1 - y$

- (b) Find the particular solution of the differential equation  $y'' + a^2y = \sec ax$

5. An L-C-R circuit with battery e.m.f 'E sin pt 'is tuned to resonance so

that  $p^2 = \frac{1}{LC}$  . If initially the current i and the charge q be zero , then

show that for small value of  $\frac{R}{L}$  , the current in the circuit at time t is

given by  $\frac{E}{2L} t \sin pt$ .

**SECTION-C**

**(10 × 2 = 20 Marks)**

6. (a) Find the eigen values and the corresponding eigen vectors of the matrix

$$\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$

- (b) Test the convergence of the series  $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^2 + 1}$

7. (a) Test the consistency of the system of equations

$$x + 2y - z = 3; 3x - y + 2z = 1; 2x - 2y + 3z = 2; x - y + z = -1,$$

and if consistent, then solve it completely.

- (b) Reduce the matrix

$$\begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

to normal form and hence find its rank.

8. (a) Discuss for what values of 'x' does the series

$$\frac{1}{2}x + x^2 + \frac{9}{8}x^3 + x^4 + \frac{25}{32}x^5 + \dots \infty, \text{ converge/diverge.}$$

- (b) Examine the convergence/diverge of the series

$$\sum_{n=1}^{\infty} [(n^3 + 1)^{\frac{1}{3}} - n]$$

9. (a) Use Demoivre's theorem to find all the roots of the equation

$$z^4 - (1-z)^4 = 0$$

- (b) Find the sum to infinity of the series

$$1 - \frac{1}{2} \cos \theta + \frac{1.3}{2.4} \cos 2\theta - \frac{1.3.5}{2.4.6} \cos 3\theta + \dots, (-\pi < \theta < \pi).$$