Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 3**

Total No. of Questions : 09

B.Tech. (Sem.–2nd)

ENGINEERING MATHEMATICS-II

Subject Code : BTAM-102 (2011 Batch)

Paper ID : [A1111]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTION TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C. have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

1. Solve the following sums :

(a) Test whether the set of vectors {(1,1,1), (1,–1,1), (3,–1,3)} are LI or LD by giving suitable reason ?

(b) Find the rank of the matrix $\begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$

(c) Reduce the matrix $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$ to diagonal form.

(d) If $x = \cos\theta + i \sin\theta$, and $y = \cos\phi + i \sin\phi$, then show that $\dfrac{x-y}{x+y} = i\tan\dfrac{\theta-\phi}{2}$

(e) Find all the values of $\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{3/4}$.

(f) Examine the conditional convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}$.

(g) Test the convergence of the series $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{-n^2}$.

(h) Show that the necessary condition for the differential equation $M\,dx + N\,dy = 0$ $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ is

(i) Find the particular solution of the equation $\frac{d^3y}{dx^3}+4\frac{dy}{dx} = \sin 2x$.

(j) Solve the equation $e^{2z-1} = 1 + i$

SECTION-B

2. (a) Obtain the general solution of the equation $y'' - 6y' + 9y = e^{3x} / x^2$, by using method of variation of parameters,

   (b) Find the complete solution of the differential $y'' - 2y' + y = x\,e^x \sin x$.

3. (a) Solve the following simultaneous differential equation

   $\frac{dx}{dt}-2y + 5x = t,\ \frac{dy}{dt} + 2x + y = 0$. Given that $x(0) = 0,\ y(0) = 0$.

   (b) Find the complete solution of the differential equation

   $(1 + x)^2y'' + (1 + x)y' + y = 2\sin \log(1 + x)$.

   by using operator method.

4. (a) Solve the differential equation $(xy^2 - e^{1/x^3})\,dx - x^2\,y\,dy = 0$

   (b) Solve the equation $y = 2px + yp^2$ where p has its usual meaning.

5. An e.m.f. $E \sin pt$ is applied at $t = 0$ to a circuit containing a capacitance C and inductance L. The current i satisfies the equation $L\frac{di}{dt}+\frac{1}{C}\int i\,dt = E \sin pt$. If $p^2 = 1/LC$ and initially the current and the charge are zero then find the current at any time t.

SECTION-C

6. (a) Use the rank method to test the consistency of the system of equations

$4x - y = 12$; $- x + 5y - 2z = 0$; $-2y + 4z = - 8$;

if consistent then solve it completely,

(b) State Cayley-Hamilton theorem. Use it to find the inverse of the matrix

$$\begin{pmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{pmatrix}$$

7. (a) Find the eigen values and the corresponding eigen vectors of the matrix

$$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

(b) Prove that eigen values of a skew hermetian matrix are either zero or purely imaginary.

8. (a) Test for what values of x the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n, \ x > 0$$

Convergences/diverges

(b) Test convergence/diverge of the series

$$\sum_{n=1}^{\infty} \left[\sqrt{(n^4+1)} - \sqrt{(n^4-1)} \right]$$

9. (a) Use Demoivre's theorem to solve the equation $(z - 1)^5 + z^5 = 0$

(b) Separate $\sin^{-1}(e^{i\theta})$ into real and imaginary parts, where θ is a positive acute angle.