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B.Tech. (Sem. - 1st / 2nd)

ENGINEERING MATHEMATICS - II

SUBJECT CODE : AM - 102 (2k4 & Onwards)

Paper ID : [A0119]

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours **Maximum Marks : 60**

Instruction to Candidates:

1) Section - A is **Compulsory**.
2) Attempt any **Five** questions from Section - B & C.
3) Select atleast **Two** questions from Section - B & C.

Section - A

Q1) *(Marks : 2 each)*

a) What do you understand by complementary function? Explain.

b) Are these vectors linearly independent?

$x_1 = (1, 2, 1), x_2 = (2, 1, 4), x_3 = (4, 5, 6), x_4 = (1, 8, -3)$

c) State Cayley - Hamilton theorem.

d) Define order and degree of an ordinary differential equation.

e) State necessary conditions for an ordinary differential equation to be exact.

f) Define directional derivative of a function.

g) State Type I and Type II errors in sampling.

h) Define Null hypothesis and critical region.

i) State Gauss divergence theorem.

j) Show that if $A = \begin{bmatrix} -1 & 2+i & 5-3i \\ 2-i & 7 & 5i \\ 5+3i & -5i & 2 \end{bmatrix}$, then iA is skew Hermitian.

Section - B

(Marks : 8 each)

Q2) (a) Solve the equations :

$$5x + 3y + 7z = 4,\ \ 3x + 26y + 2z = 9,\ \ 7x + 2y + 11z = 5$$

(b) Find the eigen-values and eigenvectors of $A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$.

Q3) (a) Prove that the necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$, to be exact is $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$.

(b) Solve : $xdy - ydx = (x^2 + y^2)dx$.

Q4) (a) Solve : $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = xe^x \sin x$

(b) $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + 4 = \log x \sin(\log x)$

Q5) (a) If an e.m.f. $E \sin \omega t$ is applied to L-C-R circuit at time t satisfies the equation $L\dfrac{d^2q}{dt^2} + R\dfrac{dq}{dt} + \dfrac{q}{c} = E \sin \omega t$. If $R = 2\sqrt{LC}$, solve the differential equation for q.

(b) A body executes damped forced vibrations given by the equation

$$\frac{d^2x}{dt^2} + 2K\frac{dx}{dt} + bx^2 = e^{-Kt} \sin \omega t.$$

Solve the equation for the cases : (i) $\omega^2 \neq b^2 - K^2$ and $\omega^2 = b^2 - K^2$.

Section - C

(Marks : 8 each)

Q6) (a) If $\vec{r} = (a \cos t, a \sin t, at \tan \alpha)$, find $\left| \frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2} \right|$ and $\left| \frac{d\vec{r}}{dt}, \frac{d^2\vec{r}}{dt^2}, \frac{d^3\vec{r}}{dt^3} \right|$.

(b) Prove that $\nabla \times (\phi \vec{a}) = \phi \nabla \times \vec{a} + \nabla \phi \times \vec{a}$.

Q7) (a) State and prove Stokes theorem.

(b) Use divergence theorem to evaluate $\iint_S \vec{F}.dS$, $\vec{F} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$, S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

Q8) (a) Find mean and variance of Poisson distribution.

(b) Find a binomial distribution for the following data :

x :	0	1	2	3	4	5
f :	2	14	20	34	22	8

Q9) (a) A sample of 20 items has mean 42 units and S.D. 5 units .test the hypothesis that it is a random sample from the normal population with mean 45 units given that $t_{0.05} = 2.09$, for 19 d.f.

(b) Write a short note on hypothesis testing and its uses.

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