Roll No. .... **Total No. of Pages : 03**

Total No. of Questions : 09

B.Tech. (Sem.–2)
ENGINEERING MATHEMATICS-II
Subject Code : AM-102 (2004-2010 Batch)
Paper ID : [A0119]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTION TO CANDIDATES :

1. **SECTION-A is COMPULSORY.**
2. **Attempt any FIVE questions SECTION - B & C.**
3. **Select at least TWO questions from SECTION - B & C.**

SECTION-A **(10 × 2 = 10 Marks)**

1. (a) Using Guass Jordan method find inverse of the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$.

(b) Prove that inverse of a unitary matrix is also a unitary matrix.

(c) Solve the differential equation $\frac{dy}{dx} + y = xy^3$.

(d) Show that necessary condition for the differential equation Mdx + Ndy = 0 to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. Is it sufficient also ?

(e) How many times a second pendulum beats in a day?

(f) If $\vec{a}$ is a vector with constant magnitude then show that $\vec{a}$ and $\frac{d\vec{a}}{dt}$ are perpendicular, provided $\left|\frac{d\vec{a}}{dt}\right| \neq 0$.

(g) If $\vec{f} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{j}$. Evaluate $\int_C \vec{f} \cdot dl$ along the curve 'C' in xy plane $y = x^3$, from the point (1, 1) to (2, 8).

(h) Find the value of 'a' so that

$\vec{f} = (ax^2y + yz)\,\hat{i} + (xy^2 - xz^2)\,\hat{j} + (2xyz - 2x^2y^2)\,\hat{k}$ is solenoidal.

(i) If on an average '1' vessel in every 10 is wrecked, find the probability that out of 5 vessels expected to arrive, at least 4 will arrive safely.

(j) A die was thrown 9000 times and a throw of 3 or 4 was observed 3,240 times. Show that die is biased.

SECTION-B **(8 Marks each)**

2. (a) Find the value of 'λ' for which the following equations have non-zero solutions :

$x + 2y + 3z = \lambda x$

$3x + y + 2z = \lambda y$

$2x + 3y + z = \lambda z.$

(b) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$.

3. (a) Solve the differential equation :

$3x^2y^3e^y + y^3 + y^2)\,dx + (x^3y^3e^y - xy)\,dy = 0$

(b) Define Clairaut's Equation. Find its general solution.

4. (a) Find the general solution of $\frac{d^2y}{dx^2} - 4y = x\sin hx$.

(b) Solve the given equation by variation of parameters method :

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + ay = \frac{e^{3x}}{x^2}.$$

5. (a) In an LCR circuit an inductance L of one henry, resistance of 6 ohm and a condensor of $\frac{1}{9}$ farad have been connected through a battery of e.m.f. '$E = \sin t$'. If $I = \theta = 0$ at $t = 0$, find charge θ and current I.

(b) At the end of three successive seconds, the distances of a point moving with simple harmonic motion from its mean position measured in the same direction are 1, 3, 4. Find the period of complete oscillations.

SECTION-C **(8 Marks each)**

6. (a) Prove that $\nabla \times (\phi \vec{f}) = \nabla\phi \times \vec{f} + \phi(\nabla \times \vec{f})$

where ϕ is a scalar point function & $\vec{f}$ is a vector point function.

(b) Evaluate $\iint_S \vec{f} \cdot \hat{n}\, ds$, where $\vec{f} = yz\hat{i} + xz\hat{j} + zy\hat{k}$ and S is that part of the surface of the sphere $x^2 + y^2 + z^2 = 1$, which lies in the first octant.

7. (a) Verify Gauss's divergence theorem for $\vec{f} = (x^3 - yz)\hat{i} - 2x^2y\hat{j} + 2\hat{k}$ taken over the cube bounded by the planes $x = 0$, $x = a$, $y = 0$, $y = a$, $z = 0$ and $z = a$.

(b) Verify Green's theorem in the xy plane for $\oint_C (xy^2 - 2xy)dx + (x^2y + 3)dy$ around the boundary 'C' of the region enclosed by $y^2 = 8x$ and $x = 2$, above x-axis.

8. (*a*) In a distribution exactly normal 10·03% of the items are under 25 kg wt. and 89.97% of the items are under 70 kg wt. What are the mean and standard deviation of the distribution?

(b) Fit a parabola to the following data :

x :	1	2	3	4
y :	0·30	0·64	1·32	5·40

9. (a) A sample of 10 boys had the following IQ : 70, 120, 110, 101, 88, 83, 95, 98, 107, 100.

Do these data support the assumption of population mean IQ of 100 at 5% level of significance ?

(b) A set of 5 similar coins is tossed 320 times and the result is

No of heads	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Test the hypothesis that data follows a binomial distribution.