Engineering Mathematics
(AM-102, Dec-07)

Note: Section A is compulsory. Attempt any five questions from section B & C taking at least two questions from each Section.

Section-A

1. a). Define linear independence of vectors

   b) Define Hermitian matrix with suitable example.

   c) Check the equation $(3x^2 + 2e^y)dx + (2xe^y + 3y^2)dy = 0$ for exactness.

   d) Fine the particular integral of the equation $4y'' - 4y' + y = e^{x/2}$

   e) Find the complementary function of the equation $y'' + 4y' + 3y = x \sin 2x$.

   f) Find $v'(t)$, given that $v(t) = (\cos t + t^2)(t\,i + j + 2k)$

   g) Evaluate $\int_C x^2 y\,ds$, where C is the curve defined by $x = 3\cos t$, $y = 3\sin t$ for the interval $0 \le t \le \pi/2$.

   h) Two dice are tossed once. Find the probability of getting an even number on the first dice.

   i) Check the correctness of the statement, "Mean of a binomial distribution is 3 and variance is 5".

   j) Explain Type I and Type II errors.

Section-B

2. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then show that $A^n = A^{n-2} + A^2 - I$, for $n \ge 3$. Hence find A^{50}.

3. Obtain the general and as well as singular solution of the non-linear equation $y = xy' + (y')^2$

4. Solve the system of equations
   $(2D - 4)y_1 + (3D + 5)y_2 = 3t + 2$, $\quad (D - 2)y_1 + (D + 1)y_2 = t$

5. A scratched elastic horizontal string has one end fixed and a particle of mass m is attached to the other. Find the equation of the motion of the particle given that l is the natural length of the string and e is its elongation due to weight mg. Also find the displacement s of particle when initially s = 0, v = 0.

Section-C

6. (a) find the normal vector and the equation of the tangent plane to the surface $z = \sqrt{x^2 + y^2}$ at the point (3, 4, 5).

   (b) Find the work done by the force $F = -xy\,i + y^2 j + z k$ in moving a particle over the circular path $x^2 + y^2 = 4$, $z = 0$ from (2, 0, 0) to (0, 2, 0).

7. Verify strokes theorem for the vector field $v = (3x - y)i - 2yz^2 j - 2y^2 z k$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 16$, $z > 0$.

8. In a distribution which is exactly normal, 12% of the items are under 30 and 85% are under 60. Find the mean and standard deviation of the distribution. (Area under normal curve for $0 \le z \le 0.38$ is 1.1750 and for $0 \le z \le 0.35$ is 1.0365)

9. Annual rainfall at a certain place is normally distributed with mean 45 cm. The rainfalls for the last five years are 48 cm, 42 cm, 40 cm, 44 cm and 43 cm. Can it be concluded that the average rainfall during the last five years is less than the normal rainfall? (Given that $t_{0.05}$ for v = 2.776)