

Time: 3hrs

BTAM - 101

M.M : 60

**Note:** Question-I is compulsory to attempt, consisting of ten short answer type questions carrying two marks each. Attempt five questions (carrying eight marks each) by selecting at least two questions each from Section -A and Section -B .

- I(a) Identify the symmetries of the curve  $r = \sin \frac{\theta}{2}$ .
- (b) If  $u = F\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ , then show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$
- (c) If  $\vec{u}$  and  $\vec{v}$  are irrotational. Vectors, then show that  $\vec{u} \times \vec{v}$  is a solenoidal vector.
- (d) Give the physical interpretation of curl of a vector point function
- (e) Evaluate:  $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z) dx dy dz$
- (f) If  $\vec{u}$  is a differentiable vector function of t of constant direction, then show that  $\vec{u} \times \frac{d\vec{u}}{dt} = 0$
- (g) If  $x = u(1+v)$  and  $y = v(1+u)$ , then find the value of  $\frac{\partial(x,y)}{\partial(u,v)}$ .
- (h) Graph the set of points whose polar co-ordinates satisfy the inequalities

$$\pi/4 \leq \theta \leq 3\pi/4, 0 \leq r \leq 1$$

- (i) Change the Cartesian integral  $\int_0^6 \int_0^y x dx dy$  into an equivalent polar integral .
- (j) What surface is represented by  $36x^2 + 9y^2 + 4z^2 = 36$ ?

### Section-A

II(a) Trace the curve  $y^2(a+x) = x^2(3a-x)$  by giving all salient features in detail.

(b) If  $\rho_1$  and  $\rho_2$  be radii of curvature at the ends of any chord of  $r = 1 + \cos \theta$  which passes through the pole, then prove that

$$\rho_1^2 + \rho_2^2 = \frac{16}{9}$$

III (a) Show the volume of the solid generated by the revolution of the cycloid

$$x = a(\theta + \sin \theta), y = a(1 - \cos \theta), 0 \leq \theta \leq \pi \text{ about the tangent at vertex is } \pi^2 a^3.$$

(b) Find the centre of gravity of the area between the curve  $y^2(2a - x) = x^3$  and its asymptote.

IV (a) If  $u = \sin^{-1}(x - y)$ ,  $x = 3t$ , and  $y = 4t^3$  then show that  $\frac{du}{dt} = 3(1 - t^2)^{-\frac{1}{2}}$

(b) State Euler's theorem and use it to prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3u \log u, \text{ whenever } u = e^{x^2 + y^2}$$

V(a) Find the volume of the largest rectangular parallelopiped that can be inscribed in

$$\text{the ellipsoid } \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

(b) Expand  $e^x \log(1 + y)$  in ascending powers of  $x$  and  $y$  upto third degree terms.

### SECTION-B

VI(a) Evaluate:  $\int_0^\infty \int_0^x x e^{-\frac{x^2}{y}} dx dy$  by changing the order of integration.

(b) Find the volume of the tetrahedron bounded by the planes

$$x = 0, y = 0, z = 0, \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, \quad a, b, c \geq 0.$$

VII(a) If  $r = \left| \vec{r} \right|$  and  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$  and  $\vec{a}$  is a constant vector, then prove that

$$\nabla \times \left( \frac{\vec{a} \times \vec{r}}{r^n} \right) = \frac{2 - n}{r^n} \vec{a} + \frac{n(\vec{a} \cdot \vec{r})}{r^{n+2}} \vec{r}$$

(b) A vector field is given by  $\vec{F} = \sin y \vec{i} + x(1 + \cos y) \vec{j}$ . Evaluate the line integral

$$\int_C \vec{F} \cdot d\vec{r} \text{ over a circular path given by } x^2 + y^2 = a^2, z = 0.$$

VIII(a) Find the directional derivative of  $f(x, y, z) = x^2 y^2 + yz^3$  at  $(2, -1, 1)$  in the direction of normal to the surface  $x \log z - y^2 = -4$  at  $(-1, 2, 1)$ .

(b) Find the area lying outside the cardioid  $r = a(1 - \cos \theta)$  and inside the circle  $r = a \sin \theta$

IX(a) State Stoke's theorem and use it to evaluate  $\int_C y dx + z dy + x dz$  where C is the curve

of intersection of the sphere  $x^2 + y^2 + z^2 = a^2$ , and the plane  $x + z = a$

(b) Verify Green's theorem for  $\oint_C [(3x^2 - 8y^2) dx + (4y - 6xy) dy]$ , where C is the

Boundary of the region bounded by the lines  $x = 0, y = 0$ , and  $x + y = 1$