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Total No. of Questions : 09

B.Tech. (Sem.–1st)

ENGINEERING MATHEMATICS-I

Subject Code : BTAM-101 (2011 & 2012 Batch)

Paper ID : [A1101]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTION TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

1. Answer briefly :

(a) Identify the symmetry of the polar curve $r = \sin\frac{\theta}{2}$.

(b) If $u = F(x-y, y-z, z-x)$, then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

(c) If $J = \frac{\partial(u,v)}{\partial(x,y)}, J' = \frac{\partial(x,y)}{\partial(u,v)}$, then show that $JJ' = 1$, where J stands for Jacobian.

(d) Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dydx$.

(e) Find the polar equation of the curve $x^2 + (y-3)^2 = 9$ given in Cartesian form .

(f) State Gauss Divergence Theorem.

(g) If $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$, then find div $\vec{F}$.

(h) Find the work done by the force field $\vec{F} = (y - x^2)\hat{i} + (z - y^2)\hat{j} + (x - z^2)\hat{k}$ over the curve $\vec{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$, $0 \le t \le 1$, from (0,0,0) to (1,1,1).

(i) Obtain the local extreme values of the function $f(x, y) = xy$.

(j) The period of a simple pendulum is $T = 2\pi\sqrt{l/g}$, find the maximum error in T due to possible error up to 1% in l and 2.5% in g.

SECTION-B

2. (a) Trace the curve $y^2(a - x) = x^2(a + x)$ by giving all salient features in detail.

(b) If ρ_1 and ρ_2 be the radii of curvature at the extremities of two conjugate diameters of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then prove that $(\rho_1)^{2/3} + (\rho_2)^{2/3} (ab)^{2/3} = a^2 + b^2$. **(4, 4)**

3. (a) Find the entire length of the Cardiode $r = a(1 + \cos\theta)$. Also show that upper half is bisected by the ray $\theta = \pi/3$.

(b) The area bounded by an arc of the curve

$x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, $0 \le \theta \le 2\pi$

and the x-axis is revolved around x-axis. Find the volume of the solid generated. **(4, 4)**

4. (a) Transform the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into polar co-ordinates.

(b) If $u = \sin^{-1}\frac{x + y}{\sqrt{x} + \sqrt{y}}$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$. **(5, 3)**

5. (a) A rectangular box open at the top is to have a volume of 32 cubic feet. Find the dimensions of the box requiring the least material for its construction.

(b) Expand $f(x, y) = \sin xy$ in ascending powers of $(x - 1)$ and $(y - (\pi/2))$ up to second degree terms. **(4, 4)**

SECTION-C

6. (a) Find the area lying inside the curve $r = a(1 + \cos\theta)$ and outside the curve $r = a$.

(b) Evaluate: $\int_0^1 \int_{x^2}^{2-x} xy\, dxdy$ by changing the order of integration. **(4, 4)**

7. (a) Prove the identity $\nabla \times (\vec{F} \times \vec{G}) = \vec{F}(\nabla \cdot \vec{G}) - \vec{G}(\nabla \cdot \vec{F}) + (\vec{G} \cdot \nabla)\vec{F} - (\vec{F} \cdot \nabla)\vec{G}$.

(b) If $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$, then evaluate $\iint_S \vec{F} \cdot \hat{N}\, ds$, where S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 1$, $z = 1$. **(4, 4)**

8. (a) Verify Stoke's theorem for the field $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, over the upper half surface of $x^2 + y^2 + z^2 = 1$ bounded by its projection on the xy-plane.

(b) Compute the line integral $\int_C (y^2 dx - x^2 dy)$ about the triangle whose vertices are (1, 0), (0, 1), (– 1, 0). **(5, 3)**

9. (a) Verify Green's theorem for $\oint_C [(3x^2 - 8y^2)\, dx + (4y - 6xy) dy]$, where C is the boundary of the region by $x = 0$, $y = 0$, $x + y = 1$.

(b) Evaluate the triple integral $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz\, dx\, dy\, dz$. **(5, 3)**