Roll No. Total No. of Pages : 03

Total No. of Questions : 09

B.Tech. (Sem.–1st)

ENGINEERING MATHEMATICS-I

Subject Code : AM-101 (2005-2010 Batches)

Paper ID : [A0111]

Time : 3 Hrs. Max. Marks : 60

INSTRUCTION TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B & C have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

1. **Write briefly :**

(a) If $x^2 = au + bv$, $y^2 = au - bv$, then prove that $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_v = \frac{1}{2}$.

(b) If $u = \log(x^2 + xy + y^2)$, then use Euler's theorem to show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2$$

(c) Express the integral $\int_0^\infty e^{-x^4} dx$ in terms of gamma function.

(d) Test the convergence/divergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$.

(e) Change the order of integration of $\int_{0}^{2a}\int_{x^2/4a}^{3a-x} f(x, y)\, dx\, dy$.

(f) Test whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$ is absolutely convergent or not.

Explain how?

(g) Use De-Moivre's theorem to express $\sin^5\theta \cos^2\theta$ in a series of sines of multiples of θ.

(h) Find the general value i^i.

(i) Verify that $f_{xy} = f_{yx}$, when $f(x, y) = \sin^{-1}(y/x)$.

(j) Obtain the real part of $\tan(A + iB)$.

SECTION-B

2. (a) Trace the curve $y^2(a + x) = x^2(3a - x)$ be giving all salient features in detail.

   (b) Find the radius of curvature at the point (r, θ) of the curve $r = a(1 - \cos\theta)$ and show that ρ^2 varies as r^2.

3. (a) Find the area included between the curves $r = a(1 - \cos\theta)$ and $r = a(1 + \cos\theta)$.

   (b) Find the surface of the solid generated by the revolution of the curve $x = a\cos^3 t$, $y = b\sin^3 t$ about y-axis.

4. (a) If $x^x y^y z^z = c$, then show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log e\, x)^{-1}$.

   (b) Find the points on the surface $z^2 = xy + 1$ nearest to the origin

5. (a) Find the centre of gravity of the arc of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ in the first quadrant.

   (b) Expand $f(x, y) = \sin x y$ in ascending powers of $(x - 1)$ and $(y - \pi/2)$ by using Taylor's theroem up to second degree terms.

SECTION-C

6. (a) Evaluate $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2+y^2)\, dx\, dy$, by changing into polar co-ordinates.

   (b) Find the volume bounded by the co-ordinate planes $x=0$, $y=0$, $z=0$ and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$.

7. (a) Find the centre and radius of the circle

   $x^2 + y^2 + z^2 - 2x - 4y - 6z - 2 = 0,\ x + 2y + 2z - 20 = 0.$

   (b) Find the equation of a cone whose semi vertical angle is $\pi/4$, has its vertex at origin and axis along the line $x = -2y = z$.

8. (a) If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then prove that

   $$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{3}{2} = \sin^2\alpha + \sin^2\beta + \sin^2\gamma$$

   (b) If $\sin^{-1}(u + iv) = \alpha + i\beta$ then prove that $\sin^2\alpha$ and $\cosh^2\beta$ are the roots of the equation

   $x^2 - (1 + u^2 + v^2)x + u^2 = 0.$

9. (a) Test for what values of x does the series

   $$\frac{x}{1.2}+\frac{x^2}{3.4}+\frac{x^3}{5.6}+\frac{x^4}{7.8}+....\infty$$

   converges/diverges.

   (b) Examine the convergence of the following series :

   (i) $\sum_{n=1}^{\infty} \frac{e^n}{1+e^{2n}}$

   (ii) $\sum_{n=1}^{\infty} \frac{1}{1+2^2+3^2+....+n^2}$