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Total No. of Questions : 09

B. Tech. (Sem.–1)
ENGINEERING MATHEMATICS-I
Subject Code : BTAM-101 (2011 Batch)
Paper ID : [A1101]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTION TO CANDIDATES :

1. **Question-1 is compulsory to attempt, consisting of ten short answer type questions carrying two marks each.**
2. **Attempt five questions (carrying eight marks each) by selecting at least two questions each from Section A and Section B**

1.a) If $w = x^2 + y^2$, $x = r - s$, $y = r + s$, then find $\frac{\partial w}{\partial r}, \frac{\partial w}{\partial s}$, in terms of r, s.

b) Write the Cartesian equivalent of the polar curve $r \cos\left(\theta - \frac{\pi}{2}\right) = 2$.

c) Find the directions in which $f(x,y) = (x^2 + y^2)/2$ increases and decreases most rapidly at the point (1,1).

d) Give the physical interpretation of gradient of a scalar point function.

e) Calculate the outward flux of the field $\vec{F} = x\hat{i} + y^2\hat{j}$ across the square bounded by the lines $x = \pm 1$ and $y = \pm 1$.

f) If $\vec{u}$ is a differentiable vector function of t of constant magnitude, then show that

$$\vec{u} \cdot \frac{d\vec{u}}{dt} = 0.$$

g) If $u = \dfrac{x+y}{1-xy}$ and $v = \tan^{-1} x + \tan^{-1} y$, then find the value of $\dfrac{\partial(u,v)}{\partial(x,y)}$.

h) Find the area enclosed by the lemniscate $r2 = 4\cos 2\theta$.

i) Change the Cartesian integral $\displaystyle\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{\sqrt{x^2+y^2}}\, dy\, dx$ into an equivalent polar integral

j) Evaluate : $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (xyz)\, dz\, dy\, dx$.

SECTION-A **(8 marks each)**

2. a) If ρ_1 and ρ_2 be the radii of curvature at the extremities of a focal chord of the parabola $y^2=4ax$, then prove that $(\rho_1)^{-2/3} + (\rho_2)^{-2/3} = (2a)^{-2/3}$.

   b) Trace the curve $r = a(1 - \sin\theta)$ by giving all its salient features in detail.

3. a) Find the surface area of the solid formed by revolving the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x–axis.

   b) Find the entire length of the cardiode $r = 1 + \cos\theta$ and further show that the upper half is bisected by the ray $\theta = \pi/3$.

4. a) State Euler's theorem and use it to prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0, \text{ whenever } u = x\phi\left(\frac{y}{x}\right) + \varphi\left(\frac{y}{x}\right)$$

b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}.$$

5. a) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimension of the box requiring least material for its construction.

b) Expand $x^2y + 3y - 2$ in powers of $(x-1)$ and $(y+2)$ by Taylor's theorem .

SECTION-B **(8 marks each)**

6. a) Evaluate : $\int_0^1 \int_x^1 \sin y^2 \, dy \, dx$ by changing the order of integration .

b) Find the volume of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = a$.

7. a) Prove the identity :

$$\nabla \times (\vec{F} \times \vec{G}) = \vec{F}(\nabla . \vec{G}) - \vec{G}(\nabla . \vec{F}) + (\vec{G} . \nabla)\vec{F} - (\vec{F} . \nabla)\vec{G}$$

b) A vector field is given by $\vec{F} = (\sin y)\hat{i} + x(1 + \cos y)\hat{j}$

Evaluate the line integral $\int_C \vec{F} . \vec{dr}$, where C is circular path

$x^2 + y^2 = a^2$, $z = 0$.

8. a) Verify Stoke's for a vector field defined by

$\vec{F} = (x^2 - y^2)\hat{i} + 2xy)\hat{j}$ taken around the rectangle bounded by the lines $x = 0$, $x = a$, $y = 0$, $y = b$.

b) Test whether the vector

$\vec{F}\ (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$, is irrotational or not ?

9. a) State Green's theorem in plane and use it to evaluate

$$\int_C (2x^2 - y^2)\,dx + (x^2 + y^2)\,dy$$

where C is the boundary of the region enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$.

b) Find the rate of change of $f(x, y, z) = xyz$ in the direction normal to the surface $x^2y + y^2x + yz^2 = 3$ at the point (1, 1, 1).