ENGINEERING MATHS-I
(AM-101, DEC-05)

Note: Section A is compulsory. Attempt any five questions from Section B and C taking at least two questions from each part.

Section-A

1. (a) Define the point of inflection for a curve and find (s) of inflection for the curve
$y = x^3 + 8x^3 - 270x$

(b) What do you understand by parametric curves? Give an example of a parametric curve involving two parameters.

(c) Using parametric equation of a circle, show that the area of circle of radius r is πr^2.

(d) Find the area of sphere generated by revolving the circle
$x^2 + y^2 = r^2$ about x axis

(e) If $x = r\cos\theta$ and $y = r\sin\theta$, Find $\frac{\partial(x,y)}{\partial(r,\theta)}$ *and* $\frac{\partial(r,\theta)}{\partial(x,y)}$

(f) Find the equation of normal line to the surface $xyz = a^3$ at $P(x_1, y_2, z_1)$

(g) Evaluate $\int_0^1 \int_0^1 (x+2)\,dy\,dx$

(h) State De Moivre's theorem and prove it for the most fundamental case.

(i) Define tangent plane to a sphere and derive the equation of tangent plane taking a general equation of the sphere.

(j) Define Beta function.

Section-B

2. Trace the polar curve
$r = a(1 - \cos\theta)$, where a is +ve constant.

3. Find the area contained between x-axis and one arch of the curve $y = \cos 3x$.

4. (a). Verify the Euler's theorem for
$f(x, y, z) = 3x^2yz + 3xy^2z + 4z^4$

(b) If $u = \sin^{-1}(x - y)$ $x = 3t$, $y = 4t^3$, find the value of $\frac{du}{dt}$

5. Use Languages method to find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $x+y+z=1$ and $xyz=1=0$

Section-C

6. Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x+4y+2z-3=0$. Also find the point of contact.

7. Using double integration, find the area enclosed between the curve $y^2=x^3$ and $y=x$

8. Test the following series for uniform convergence $\sum \frac{\cos n^x}{n^3}$ for $\pi<x<2\pi$.

9. If $u = \log\left(\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right)$, prove that (i) $\sinh(u) = \tan\theta$ (ii) $\tanh u = \sin\theta$