**Roll No.** ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 03**

**Total No. of Questions : 09**

**B.Tech. (Sem.–1st)**

# ENGINEERING MATHEMATICS-I

**Subject Code : BTAM-101 (2011 & 2012 Batch)**

**Paper ID : [A1101]**

**Time : 3 Hrs.** **Max. Marks : 60**

**INSTRUCTION TO CANDIDATES :**

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

## SECTION-A

1. a) Find asymptotes, parallel to axes, of the curve :

$$x^2 y^2 - xy^2 - x^2y + x + y + 1 = 0.$$

b) Write a formula to find the volume of the solid generated by the revolution, about $y$-axis, of the area bounded by the curve $x = f(y)$, the $y$-axis and the abscissae $y = a$ and $y = b$.

c) What is the value of $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)}$ ?

d) If an error of 1% is made in measuring the length and breadth of a rectangle, what is the percentage error in its area?

e) Find the equation of the tangent plane to the surface

$$z^2 = 4(1 + x^2 + y^2) \text{ at } (2, 2, 6).$$

f) What is the value of $\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$

g) Give geometrical interpretation of $\int_0^1 \int_0^{1-x} dxdy$.

h) Show that the vector field $\vec{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$ is irrotational.

i) What is the value of $\nabla \times (xy\hat{i} + yz\hat{j} + zx\hat{k})$ ?

j) State Stoke's theorem.

**SECTION-B**

2. Trace the following curves by giving their salient feature:

   a) $x^3 + y^3 = 3axy$.

   b) $r = a(1 + \cos\theta)$ (4,4)

3. a) Find the perimeter of the cardioid $r = a(1-\cos\theta)$.

   b) Find the area bounded by two parabolas $y^2 = 4x$ and $x^2 = 4y$. (4,4)

4. a) If $u = \frac{y}{z} + \frac{z}{x}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.

   b) State Euler's theorem for homogeneous functions and apply it to show that

   $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3 \tan u$$

   where $\sin u = \frac{x^2y^2}{x+y}$ (4,4)

5. a) Find points on the surface $z^2 = xy + 1$ nearest to the origin.

   b) Find percentage error in the area of an ellipse if one percent error is made in measuring its major and minor axes. (4,4)

**SECTION-C**

6\. a) Evaluate the following integral by changing the order of integration :

$$\int_0^3 \int_1^{\sqrt{4-x}} (x+y)\,dxdy$$

b) Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$ (4,4)

7\. a) Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point $(1, 2, -1)$.

b) If $\vec{F} = (x + y + 1)\,\hat{i} + \hat{j} - (x + y)\,\hat{k}$, show that $\vec{F}$ . curl $\vec{F} = 0$.

(4,4)

8\. a) Compute the line integral $\int_C (y^2dx - x^2dy)$, where C is the boundary of the triangle whose vertices are (1,0), (0,1) and (-1, 0).

b) Compute $\int_S \vec{F} \cdot \hat{N} ds$, where $\vec{F} = 6z\hat{i} - 4\hat{j} + y\hat{k}$ and S is the portion of the plane $2x + 3y + 6z = 12$ in the first octant. (4,4)

9\. State Gauss Divergence theorem and verify it for

$\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ taken over the rectangular parallelopiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c.$ (8)