B. TECH.

AM-101 : ENGG. MATHEMATICS - I

Q.1 (i) Find the general value of Log(-2)

(ii) Find the Equation of Cone with vertex at origin and which passes through the curve

$x^2+y^2=4, \ z=2$

(iii) If $z = x^2y, \ x^2+xy+y^2 = 1$ find $\frac{dz}{dx}$

(iv) Find the Equation of tangent plane to the surface $2x^2+y^2+2z = 3$ at the point (2, 1, -3)

(v) Find the radius of curveture at the origin for the curve $y2 = x2\left(\frac{a+x}{a-x}\right)$, a u any the Constant.

(vi) Discuss the Convergence of $\sum_{n=1}^{\infty} \left(\frac{n+1}{3n}\right)^n$

(vii) Find the entire length of the cardiod $r = a(1- \cos\theta)$

(viii) Change the order of integration $\int_0^a \int_y^a f(x,y)dxdy$

(ix) Define Gamma function $\sqrt{n}$ & prove that $\sqrt{(n+1)} = n\sqrt{(n)}$

(x) If $x=r\cos\theta, \ y=r\sin\theta$ find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

PART - A

Q.2 (a) Sketch the curve $y=x^3-6x^2+9x+6$.

(b) Prove that radius of curveture at any point p(x,y) on the Ellipse $x=a\cos\theta$, $y=b\sin\theta$ is $\frac{a^2b^2}{p^3}$ where p is length of perpendicular from the centre of the Ellipse on the tangent at P.

Q.3 (a) If $u = \tan^{-1}\frac{y^2}{x}$

Show that, $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\sin 2u \sin^2 u$.

(b) If Z is function of x and y and u, v be two other variables such that $u = lx+my$, $v = ly-mx$ Show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2+m^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$.

Q.4 (a) Find Taylor series of $x^2-3y^2- 9y - 9x+26$. of maximum order a bent the point (2, 2)

(b) Prove that of all rectangular paralleloped with same volume, the cube as the least Surface.

Q.5 (a) Find the length of the arc of parabola $x^2=49y$ extending from the vertex to one Extremely of the latus rectum.

(b) Find the volume of the solid obtained by revalving the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} =1$ about x axis.

PART B

Q.6 (a) Find the Equation of right circular cylinder of radius 2, whose axis lies along the line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-2}{5}$

(b) Find the Equation of splere lianing the circle $x^2+y^2+z^2+10y-4z-8 = 0$, $x+y+z = 3$ as one bits great circles.

Q.7 (a) Find the area bounded by the parabola $y = x^2$ and the line $y = 2x+3$

(b) Evaluate $\iint \sqrt{a^2-x^2-y^2}\,dxdy$ over the semicircle $x^2 + y^2 = ax$ in the positive quadrent by changing into polar coordinates.

Q.8 (a) Discuss the Convergence of series

$$1+\frac{3}{7}x+\frac{3.6}{7.10}x^2+\frac{3.6.9}{7.10.13}x^3+--\infty., \ x>0$$

(b) Discuss convergence the tollouins series

(i) $\sum_{n=1}^{\infty}(\sqrt{x^2+1}-n)$ (ii) $\sum_{n=1}^{\infty}\frac{(-1)^n}{x^2+1}$

Q.9 (a) Sum the series :

$$\sin\alpha + x\sin(\alpha+\beta)+\frac{x^2}{2}\sin(\alpha+2\beta)+----.$$

(b) Prove that roots of the Equation $(x-1)^n = x^n$

(n being a the integer) are $\frac{1}{2}\left(1+i\cot\frac{r\pi}{n}\right)$

Where r = 1, 2, ----- n-1.