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B.Tech. (Sem. - 1st)

ENGINEERING MATHEMATICS - I

SUBJECT CODE : AM - 101 (2K4 & Onwards)

Paper ID : [A0111]

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours **Maximum Marks : 60**

Instruction to Candidates:

1) Section - A is **Compulsory**.
2) Attempt any **Five** questions from Section - B & C.
3) Select at least **Two** questions from Section - B & C.

Section - A

Q1) ***(Marks : 2 Each)***

a) Find the entire length of the cardiode

$$r = a(1 + \cos\theta)$$

b) Use Euler's theorem to show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u\log u,$$

where $u = e^{x^2 + y^2}$.

c) If $x = r\cos\theta$, $y = r\sin\theta$, then show that

$$\frac{\partial(r,\theta)}{\partial(x,y)} = \frac{1}{r}.$$

d) Find the percentage error in the area of an ellipse when an error of +1 percent is made in measuring the major and minor axes.

e) For what value(s) of k will the plane

$x - 2y - 2z = k$ touch the sphere

$x^2 + y^2 + z^2 - 2x + 4y - 6z + 5 = 0.$

f) Evaluate $\int_0^1\int_0^{\sqrt{1-y^2}} (x^2 + y^2)\,dx\,dy$, by changing to polar co-ordinates.

g) State cauchy root test and use it test the convergence of the series:

$$\sum\left(\frac{n}{n+1}\right)^{n^2}.$$

h) Examine the convergence of

$$\frac{1}{\log 2}-\frac{1}{\log 3}+\frac{1}{\log 4}-\frac{1}{\log 5}+\ldots\ldots\ldots$$

i) If $\sin(A + iB) = x + iy$, then prove that

$$\frac{x^2}{\sin^2 A}-\frac{y^2}{\cos^2 A}=1.$$

j) Find all the values of $(1 - i)^{1+i}$.

Section - B

(Marks : 8 Each)

Q2) (a) Trace the curve $y^2(2a - x) = x^3$, by giving all its features in detail.

(b) Prove that the radius of curvature of the curve $r^n = a^n \cos n\theta$, $n = 1, 2, \ldots$ at any point (r, θ) is $\frac{a^n}{(n+1)r^{n-1}}$.

Q3) (a) Find the area of one loop of the curve $x(x^2 + y^2) = a(x^2 - y^2)$.

(b) Obtain the volume of the spindle-shaped solid generated by revolving the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis.

Q4) (a) If $u = \log_e(x^3 + y^3 + z^3 - 3xyz)$, then show that

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=-\frac{9}{(x+y+z)^2}$$

(b) Transform the Laplacian equation $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=0$, into polar form.

Q5) (a) Expand $f(x, y) = x^2y + 3y - 2$ in ascending powers of $(x - 1)$ and $(y + 2)$ using Taylor's theorem for several variables.

(b) Prove that a rectangular solid of maximum volume which can be inscribed in a sphere is a cube.

Section - C

(Marks : 8 Each)

Q6) (a) A plane passes through a fixed point (a, b, c) and cuts axes in A, B and C. Show that locus of the center of the sphere OABC is

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2.$$

(b) Find the equation of the cone whose vertex is (1, 2, 3) and guiding curve is the circle.
$x^2 + y^2 + z^2 = 4, \quad x + y + z = 1.$

Q7) (a) Evaluate the integral

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\, dy\, dx.$$

by changing the order of integration.

(b) Establish the result.

$$\int_a^b (x-a)^{m-1}(b-x)^{n-1}\,dx = (b-a)^{m+n-1}\beta(m,n),$$

m, n, a, b are positive constants.

Q8) (a) Discuss the Convergence/Divergence of the series

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^{3/2}}$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$

(b) Find the radius and interval of convergence of the series.

$$\sum \frac{(3x+1)^{n+1}}{2n+2}$$

Further, for what values of x (if any) does the series converges

(i) absolutely (ii) conditionally.

Q9) (a) Find the sum of the series:

$$\sin^2\theta - \frac{1}{2}\sin 2\theta \sin^2\theta + \frac{1}{3}\sin 3\theta \sin^3\theta ----\infty.$$

(b) Use De-Moivre's theorem to solve the equation $(x-1)^5 + x^5 = 0$.