Roll No. ....

Total No. of Questions : 09] **[Total No. of Pages : 03**

B. Tech. (Sem. - 1st/2nd)

ENGINEERING MATHEMATICS - I

SUBJECT CODE : AM - 101 (2K4)

Paper ID : [A0111]

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours **Maximum Marks : 60**

Instruction to Candidates:

1) Section - A is **Compulsory**.
2) Attempt any **Five** questions from Section - B & C.
3) Select at least **Two** Questions from Section - B & C.

Section - A

Q1) *(2 Marks Each)*

a) Find the equation of normal to the surface : $x^2 + y^2 + z^2 = a^2$.

b) Examine the convergence of $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+.....$

c) Define a homogeneous function.

d) If $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, then what is $\beta\left(\frac{1}{2},\frac{1}{2}\right)$?

e) M.I. of rectangular lamina about its side is =?

f) Name the curve represented by : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=\frac{2z}{c}$

g) If $(3 + x)^3 - (3 - x)^3 = 0$, then prove that $x = 3i \tan \frac{r\pi}{3} r = 0, 1, 2,$.

h) State DeMoivre's theorem.

i) What is $i^i = ?$

j) State Ratio test.

Section - B

(8 Marks Each)

Q2) (a) Use method of Lagrange's to find the minimum value of $x^2 + y^2 + z^2$, given that $xyz = a^3$.

(b) Expand $e^x \log(1 + y)$ up to six terms of the Taylor series in the neighborhood of (0,0).

Q3) (a) If $u = x + y + z$, $uv = y + z$, $uvw = z$ show that $\dfrac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$.

(b) if $u = \tan^{-1} \dfrac{x^3 + y^3}{x + y}$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 2 \sin 2x$.

Q4) (a) Trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

(b) Find the curvature and radius of curvature of the curve :

$x = \theta - \sin\theta, y = 1 - \cos\theta$.

Q5) (a) Show that the length of an arc of the cycloid :

$x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ is $8a$.

(b) Find the volume generated by revolving the ellipse : $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ about the x-axis.

Section - C

(8 Marks Each)

Q6) (a) Find the equation of the cone whose vertex is (1,2,3) and which passes through the circle $x^2 + y^2 + z^2 = 4$, $x + y + z = 1$.

(b) Find the centre and radius R of the circle $x^2 + y^2 + z^2 - 2y - 4z = 11$, $x + 2y + 2z = 15$.

Q7) (a) Change the order of integration in $I = \int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$ and hence evaluate it.

(b) Find the volume of the tetrahedron bounded by the coordinate axes and the plane $x + y + z = a$ by triple integration.

Q8) (a) Sum the series : $\sin\alpha+\sin(\alpha+\beta)+\sin(\alpha+2\beta)+\ldots.+\sin(\alpha+\overline{n-1}\beta)$.

(b) If $x + iy = \cosh(u + iv)$ show that $\dfrac{x^2}{\cosh^2 v}+\dfrac{y^2}{\cosh^2 u}=1$.

Q9) (a) Find the interval of convergence of the series $x-\dfrac{x^2}{\sqrt{2}}+\dfrac{x^3}{\sqrt{3}}-\dfrac{x^4}{\sqrt{4}}+\ldots.\infty$.

(b) Test the convergence of the series :

(i) $\sqrt{x^3+1}-x$.

(ii) $\dfrac{1}{1.3}+\dfrac{2}{3.5}+\dfrac{3}{5.7}+\ldots\infty$.