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Total No. of Questions : 09

B.Tech. (2011 Onwards) (Sem.–1)

ENGINEERING MATHEMATICS – I

Subject Code : BTAM-101

Paper ID : [A1101]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**
5. **Symbols used have their usual meanings. Statistical tables, if demanded, may be provided.**

SECTION-A

1. Solve the following :

a) Find the radius of curvature of the parabola $y^2 = 4ax$ at any point (x,y).

b) Find the length of the arc of the curve $y = \log \sec x$ from $x = 0$ to $x = \pi/3$.

c) If $u = exp(x^y)$, then find $\frac{\partial^2 u}{\partial y \partial x}$.

d) If $z = xy^2 + x^2y$, $x = at^2$, $y = 2at$, then find $\frac{dz}{dt}$.

e) Find the percentage error in calculating the area of a rectangle, when an error of 3% is made in measuring each of its sides.

f) Evaluate $\int_0^3 \int_0^1 (x^2 + 3y^2)dydx$.

g) Find the magnitude of velocity and acceleration of a particle which moves along the curve $x = 2 \sin 3t$, $y = 2 \cos 3t$, $z = 8t$ at any time $t > 0$.

h) Show that $\overline{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)\,j + (3xy - 2xz + 2z)k$ is solenoidal.

i) State Green's theorem in plane.

j) State Gauss divergence theorem.

SECTION-B

2. a) Trace the cardioid $r = a(1 - \cos\theta)$.

   b) Trace the cissoid $y^2(2a - x) = x^3$

3. a) Find the area bounded by the ellipse $b^2x^2 + a^2y^2 = a^2b^2$, $(a > b)$.

   b) Find the volume of the solid generated by the revolution of the loop of the curve $x = t^2$, $3y = 3t - t^3$ about the x axis.

4. a) If $u = \dfrac{x^3y^3}{x^3 + y^3}$, then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3u$.

   b) If $u = x^2 - y^2$, $v = 2xy$ and $x = r\cos\theta$, $y = r\sin\theta$ then find $\dfrac{\partial(u,v)}{\partial(r,\theta)}$.

5. Using Lagrange's method of undetermined multipliers, show that the rectangular solid of maximum value that can be inscribed in a sphere is a cube.

SECTION-C

6. a) Change the order of integration of $\int_0^1\int_x^{\sqrt{x}} f(x,y)\,dydx$.

   b) Using the transformation $u = x - y$, $v = x + y$, evaluate $\iint \cos\left(\dfrac{x-y}{x+y}\right) dxdy$ over the region bounded by the lines $x = 0$, $y = 0$, $1 = x + y$.

7. a) Evaluate $\iiint x^2yz\,dxdydz$ over the region bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y + z = 1$.

   b) Show that $\nabla\left(\dfrac{\overline{a}\cdot\overline{r}}{r^n}\right) = \dfrac{\overline{a}}{r^n} - \dfrac{n(\overline{a}\cdot\overline{r})}{r^{n+2}}\overline{r}$, where $\overline{r} = xi + yj + zk, r = |r|$ is a constant vector.

8. a) Evaluate $\iint_S (yz\,dydz + xz\,dzdx + xy\,dxdy)$ over the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the positive octant.

   b) Find work done in moving a particle in the force field $\overline{F} = 3x^2i + (2xz - y)j + zk$ along the curves $x^2 = 4y$ and $3x^3 = 8z$ from $x = 0$ to $x = 2$.

9. Verify Gauss divergence theorem for $\overline{F} = 4xzi - y^2j + yzk$ over the cube $x = 0$, $x = 1$ $y = 0$, $y = 1$, $z = 0$, $z = 1$.