

Roll No.

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Total No. of Pages : 02

Total No. of Questions : 09

**B.Tech.(CE)/(ECE)/(EE)/(Electrical & Electronics)/
B.Tech. (Electronics & Computer Engg.)/
B.Tech. (Electronics & Electrical)/(ETE) (2011 Onwards)
B.Tech.(Electrical Engg. & Industrial Control) (2012 Onwards)
B.Tech.(Electronics Engg.) (2012 Onwards)
(Sem.-3)**

ENGINEERING MATHEMATICS – III

Subject Code : BTAM-301

Paper ID : [A1128]

Time : 3 Hrs.

Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.
3. SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.

SECTION-A

1. Write briefly :

- a) State and prove Second Shifting property for Laplace Transform.
- b) Find inverse Laplace Transform of $\frac{2(s+1)}{(s^2+2s+2)^2}$.
- c) Find Laplace Transform of $t^2 \cdot \sinh t$.
- d) State Rodrigue's formula for Legendre polynomials.
- e) Using $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$, show that $J_{-1/2} = \sqrt{\frac{2}{\pi x}} \cos x$, where $J_n(x)$ is a Bessel function of First kind.
- f) By eliminating the arbitrary function from $z = xy + f(x^2 - y^2)$, obtain a partial differential equation.
- g) Find solution of homogeneous partial differential equation $2r - 5s + 2t = 0$.
- h) If $f(z)$ is analytic and $\text{Im } f(z)$ is constant then show that $f(z)$ is constant.
- i) Show that $\oint_C \frac{dz}{(z^2+4)^2} = \frac{\pi}{16}$, where $C : |z-i| = 2$.
- j) Using the Residue theorem, evaluate $\oint_C \frac{e^z}{(z+1)^n} dz$, where $C : |z| = 2$.

SECTION-B

2. A periodic function of period 2 is defined as $f(x) = 1 + x$, $-1 < x < 1$. Obtain the Fourier series expansion of $f(x)$ and hence show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

3. Using Laplace Transform, solve the given initial value problem

$$y'' + 6y' + 9y = 8te^{2t}, y(0) = 0, y'(0) = -1.$$

4. Find the power series solution about the point $x_0 = 2$ of the equation

$$y'' + (x-1)y' + y = 0.$$

5. Find the general solution of the partial differential equation $(x + y^2)p + yq = z + x^2$.

6. Solve the partial differential equation $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$, where $u(x, 0) = 4e^{-x}$ by the method of separation of variables.

SECTION-C

7. For Legendre polynomials $P_n(x)$, show that $\int_{-1}^1 P_m(x)P_n(x)dx = \begin{cases} 0, & m \neq n. \\ \frac{2}{2n+1}, & m = n. \end{cases}$

8. a) For Bessel's function $J_n(x)$, show that $J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 + \dots) = 1$.

- b) Determine the angle of rotation at the point $z = (1 + i)/2$ under the mapping $w = z^2$. Find its scale factor also.

9. Find all possible Taylor's and Laurent series expansions of the function

$$f(z) = \frac{1}{(z+1)(z+2)^2} \text{ about the point } z = 1.$$