Roll No. **Total No. of Pages : 03**

Total No. of Questions : 09

B.Tech.(CSE/IT) (Sem.–4)

MATHEMATICS – III

Subject Code : CS-204

Paper ID : [A0495]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTION TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

1. Write briefly :

a) Define the order of a numerical method for the solution of the initial value problem $y' = f(x,y)$, $y(x_0) = y_0$.

b) Find the residue at $z = 0$ of the function $f(z) = z\cos(1/z)$.

c) Why is the classical Runge-Kutta method of fourth-order, the most commonly used method for solving the first order initial value problems.

d) Show that the function $f(z) = \sin z$ is unbounded.

e) Classify the partial differential equation $2u_{xx} + 3u_{yy} - u_x + 2u_y = 0$.

f) Define analytic function. Give an example of a function which is not analytic.

g) Define conformal mapping. Is $w = z^2$ a conformal map?

h) Write one dimensional heat conduction equation and associated conditions.

i) State Cauchy-Residue theorem.

j) Write the bound on the error of Taylor series method.

SECTION-B

2. i) Find the volume of the prism whose base is the triangle in the xy-plane bounded by the x-axis and the lines $y = x$ and $x = 1$ and whose top lies in the plane $z = f(x,y) = 3 - x - y$.

   ii) A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at a 45° angle at the center of the cylinder. Find the volume of the wedge.

3. i) Define a harmonic function. Show that the function $u(x,y) = x^4 - 6x^2y^2 + y^4$ is harmonic. Also, find the corresponding analytic function.

   ii) Obtain the Taylor or Laurent series which represents the function $f(z) = \dfrac{1}{(1+z^2)(z+2)}$ for $1 < |z| < 2$.

4. i) Show that the transformation $w(z + i)^2 = 1$ maps inside the circle $|z| = 1$ in the z plane on the exterior of the parabola.

   ii) Using calculus of residues, evaluate the real integral

$$\oint_C \frac{\sin^2\theta}{5-4\cos\theta}dz, \quad C:|z|=1.$$

5. i) Using contour integration, evaluate the real improper integral

$$I = \int_0^\infty \frac{\sin ax}{x(x^2+b^2)}, a > 0, b > 0.$$

   ii) If $f(\zeta) = \oint_C \dfrac{3z^2+7z+1}{z-\zeta}$, where C is the circle $x^2 + y^2 = 4$, find the values of $f(3)$, $f'(1-i)$ and $f''(1-i)$.

6. i) Using the method of separation of variables, solve the parabolic partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = 16\frac{\partial u}{\partial y}$$

ii) Obtain the approximate value of $y(1.3)$ for the initial value problem

$$y' = -2xy^2,\ y(1) = 1$$

using Euler's method with step size $h = 0.1$. Compare with the exact solution $y = 1/x^2$.

SECTION-C

7. i) Solve the boundary value problem $xy'' + y = 0$, $y(1) = 1$, $y(2) = 2$ by second order finite difference method with $h = 0.25$.

   ii) Solve the initial value problem $y' = x(y - x)$, $y(2) = 3$ in the interval [2, 2.4] using the classical Runge-Kutta fourth-order method with step-size $h = 0.2$.

8. Solve $u_{xx} + u_{yy} = 0$ numerically under the boundary conditions

$$u(x,0) = 2x,\ u(0,y) = -y,$$

$$u(x,1) = 2x - 1,\ u(1, y) = 2 - y,$$

using square mesh of width $h = 1/3$.

9. The temperature distribution $u(x,t)$ in a thin, homogeneous semi-definite bar can be modelled by the initial boundary value problem

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, 0 < x < \infty, t > 0$$

$$u(x,0) = f(x),\ x > 0;\ u(0,t) = 0,\ t > 0.$$

Find the temperature distribution $u(x,t)$, $t > 0$, $0 < x < \infty$.